THE SYNTAX OF MODIFIED NUMERALS AND THE SEMANTICS OF DERIVED DEGREES
Workshop “The pragmatics of quantifiers: implicature and presupposition – experiment and theory”, ZAS Berlin, June 6-7, 2018

1. INTRODUCTION

The so-called “modified numerals” have mostly been analyzed as involving composition with the cardinal:

(1) a. [[more than ten] books] comparative numeral
b. [[at least ten] books] adverbial numeral
c. [[about ten] books] prepositional numeral

Compelling evidence can, however, be provided against this view and for a different structure (Geurts and Nouwen 2007, Arregi 2010, Ionin and Matushansky [in press], etc.):

(2) a. [more than [NP ten books]] comparative numeral
b. [at least [NP ten books]] adverbial numeral
c. [about [NP ten books]] prepositional numeral

Needless to say this requires completely different composition.

This talk:
➢ syntactic evidence for (2)
➢ some semantic consequences and possible treatments
➢ questions for further discussion

2. THE SYNTAX OF MODIFIED NUMERALS

Evidence against the accepted view: case-marking and word order

2.1. Case-marking

The labels in (1) and (2) reflect the fact that approximation can be achieved by a variety of syntactic means

Prepositions are known to assign case, as are comparative markers. In (1) they should assign case to the cardinal, in (2), to the entire NP

Russian demonstrates the latter: the genitive case assigned by the comparative bol’še ‘more’ and by the preposition okolo ‘about’ surfaces also on the noun:

(3) a. Maša kupila dva šará. unmodified numeral
   Masha bought two.ACC balloon.PAUC
   ‘Mary bought two balloons.’

b. Maša kupila bol’še dvux šarov. comparative + numeral
   Mary buy.PAST.F more two.GEN balloon.PL.GEN
   ‘Mary bought more than two balloons.’

c. Maša kupila okolo dvux jaščikov knig. P + numeral
   Mary buy.PAST.F about two.GEN box.PL.GEN book.PL.GEN
   ‘Mary bought about two boxes of books.’

Acknowledgments: This presentation draws upon joint work with Tania Ionin, Eddy Ruys and Joost Zwarts, none of whom should be held responsible by the use I’m making of it here.
In German the situation is more difficult to diagnose:

(4) Fischels Verschwinden gegen ein-en Monat nach Ostern Plank 2004
Fischel's disappearance towards one-MSG.ACC month after Easter
Fischel's disappearance at approximately one month after Easter.

According to Plank, case assignment by the preposition is only detectable where there is no external case assigner (which overrides the case assigned by the preposition, see also Pankau 2018)

2.2. Word order

Arregi 2010: in Hebrew and in Basque, ‘one’ is linearized on the other side of the NP than all other cardinals. Showing this for Hebrew:

(5) a. Dani kana sefer **exad.**
Dani buy.PAST book one
‘Dani bought one book.’

b. Dani kana **shney** sfarim.
Dani buy.PAST two book.PL
‘Dani bought two books.’

The modifiers, however, remain in the same position:

(6) a. Dani kana yoter **mi-sefer** exad.
Dani buy.PAST more from-book one
‘Dani bought more than one book.’

b. Dani kana yoter **mi-shney** sfarim.
Dani buy.PAST more from-two book.PL
‘Dani bought more than two books.’

(7) a. Dani kana lefaxot sefer **exad.**
Dani buy.PAST to-less book one
‘Dani bought at least one book.’

b. Dani kana lefaxot **šloša** sfarim.
Dani buy.PAST to-less three book.PL
‘Dani bought at least three books.’

Basque provides exactly the same arguments, plus also for prepositional numerals:

In Hebrew, **around one kilogram** is infelicitous, a bare NP should be used instead (**around kilogram**)

Kilogram one sugar.ABS around buy 3.ABS-have-1SG.ERG
‘I have bought around/about one kilogram of sugar.’

b. Hiru kilo azukre inguru erosi d-u-t.
Three kilogram sugar.ABS around buy 3.ABS-have-1SG.ERG
‘I have bought around/about three kilograms of sugar.’

The cardinal does not form a constituent with the “modifier”, the entire NP does

2.3. Indefinite measures

Matushansky and Zwarts 2017: there is no cardinal in prepositional measures:

(9) a. around a pound
b. between a kilometer and a mile
And only the NP for the preposition to compose with in singular prepositional measures in languages without articles:

(10) około litra (vodki)
around liter.GEN vodka.GEN
around a liter (of vodka)

Needless to say, the same is true for comparative numerals:

(11) a. more than a pound but less than a kilo of flour
b. bol'se litra (vodki)
more liter.GEN vodka.GEN
more than a liter (of vodka)

For these no one would object to the [more than/around [a measure]] grouping

Two potential solutions for prepositional numerals:

- measure phrases denote degrees, modified numerals behave like measure phrases, so cardinals are degrees (cf. Kennedy 2013, 2015, Rothstein 2013, 2016, 2017)
- measure phrases denote degrees, modified numerals behave like measure phrases, so any quantized indefinite NP can be a degree (Matushansky and Ruys 2015a, b, Matushansky and Zwarts 2017)

Guess what I choose

The list of my reasons for doing so is long. Treating cardinals as degrees is extremely difficult to reconcile with the fact that complex cardinals are linguistically composed expressions (see Rothstein 2017 for an attempt and Ionin and Matushansky [in press] to see how it fails and why complex cardinals are linguistic). Any quantized indefinite NPs can function as measure phrases given the appropriate context (This series is seven books long). Quantized indefinite NPs may show the syntax of measure phrases (in Slavic languages, for instance). Measure nouns themselves, being relational, do not behave as degrees (Ruys 2017, Matushansky and Ruys 2012, 2014, Matushansky, Ruys and Zwarts 2017). Combining cardinals with NPs becomes very complicated if cardinals are degrees (you need a covert many). And there is good evidence that anything can be a derived degree (Matushansky and Ionin 2014).

3. **ON THE SEMANTICS OF “PREPOSITIONAL NUMERALS”**

Matushansky and Zwarts 2017: what do “prepositional numerals” tell us about measurement?

(12) a. Don’t touch the steering wheel if you have drunk over five glasses of wine.
b. I ate around a pound of jam.
c. The mass of the meteorite was estimated at under 66 tons.
d. I was swimming between a kilometer and a mile four days a week.

Our goal: a minimum of difference between the spatial and degree uses of prepositions

(13) a. The picture is over the mantel.
over expresses a vertical relation between two material objects in 3D space
b. I ate over a pound of jam.
over expresses a vertical relation between two abstract containers in 1D space
Prior work: the semantics and pragmatics of *up to* (Nouwen 2008, 2010, Schwarz, Buccola and Hamilton 2012, Blok 2013, 2016a, b): connection to the directional preposition exploited, but not derived

Our proposal:

| There is no such thing as degrees, they are entities in a 1D space |

The core of the proposal: measure nouns denote **abstract containers** located in a vertically oriented half-open **one-dimensional space**.

Consequences:

- measure phrases denote entities and can therefore combine with prepositions
- **algebra of scalar addition and multiplication**, i.e., the scalar structure, follows from the properties of one-dimensional space
- the interpretation of spatial prepositions is **unchanged**
- constraints on the choice of prepositions are explained

The bigger picture: reconstructing degrees as entities in concrete 1D space without the need to postulate a special semantic type or sort

Problems: prepositional numerals are not syntactically PPs! Corver and Zwarts 2006, Pankau 2018: long list of differences with argument PPs

Matushansky and Zwarts 2017: their internal syntax is that of PPs, it is their external syntax that is not, but what is relevant externally is their semantics

Important: Matushansky and Zwarts 2017 only talk about PP measures, not all “prepositional numerals”

3.1. The concept of an abstract container

Stereotypical properties of **concrete containers**:

- **verticality**: a container must be vertical to contain substances
- **measurement**: a container can map different substances to the same volume unit

Natural properties of **abstract containers**:

- conceptualized as one-dimensional
- no distinction between container and content (due to one-dimensionality)
- generalized to all quantities (weight, length, …)
- share one natural zero point (the ”ground”, cf. Nouwen 2016), differ in height
- abstract containers can be stacked on top of each other
- two abstract containers with the same height are indistinguishable unless stacked

3.2. Available spatial building blocks (simplified)

Spatial building blocks in vector-space semantics (Zwarts and Winter 2000)

Two types in addition to *e* and *t*:

- type **p** of points
- type **v** of vectors, represent relations between points

Functions in spatial semantics:

- LOC maps an entity to its spatial boundary* (type ⟨p, t⟩)
- a preposition maps a boundary to a particular set of vectors (type ⟨v, t⟩)
- LOC maps a set of vectors to the set of entities that are located at those vectors (type ⟨e, t⟩)
The syntax of modified numerals and the semantics of derived degrees

(* the only adjustment necessary to the vector space semantics, notational variant for 3D, but crucial for 1D, part of the general schematization/idealization operations in spatial language, cf. Herskovits 1986)

(14) Vector-space scheme for *over*

[Diagram of vector-space scheme for *over*]

If measures are one-dimensional, nothing needs to be changed in the semantics of *over*

(15) \[ \langle e, t \rangle \]

one such entity

\[ \text{EX} \langle e, t \rangle \]

the set of entities whose top is given by these vectors

\[ \text{LOC} \langle v, t \rangle \]

the set of vectors whose endpoint is higher than that top

\[ \langle p, t \rangle \]

the (singleton) set of points corresponding to the top of that entity

\[ \text{LOC} \]

one liter entity

\[ \langle e, t \rangle \]

the set of liter entities

The question remains, of course, why the set of relevant prepositions is restricted in each language:

- to vertical (*over*) or dimension-neutral (*between*) prepositions: because containers are vertically oriented (see Matushansky and Zwarts 2017)
- to a subset of these: normally PPs cannot function as arguments, so the existential conversion in (15) is not normally available. We restrict it to some prepositions

How is it different from simply modifying the meaning of the relevant prepositions so that they can apply to degrees?

Answer: there are independent reasons to make measure nouns a sort of abstract entities

3.3. Summary

If measure nouns denote one-dimensional containers (abstract entities), it is unsurprising that they are nouns and the scalar structure can be derived from spatial structure

The price is some level of abstraction

The advantage is unification

4. Derived degrees

What happens when ordinary NPs start functioning as differentials? Are they then degrees?
(16) a. This series is **two movies/three books/seven cartoons** longer?
    b. The building is **two floors** taller.
    c. The pirates were now richer by **some loot and a dog**.

The claim in the literature is that any comparative adjective allows some sort of a differential:

(17) a. You can be **0.07 Einsteins more intelligent**.
    b. That pyramid was **seven acrobats higher**.

(17a) is uninteresting: it involves a (non-conventional, yet accommodated) unit of measure. (17b), on the other hand, is like (16): there a regular NP functions as a unit of measure.

Examples like (17a) are restricted by conventions on measuring units. (17b) and its ilk seem to be constrained by the predicates involved.

And of course, any cardinal-containing NP can function as a differential with **more** and **less**:

(18) a. There were 20 people more in the room.
    b. There were more than 20 people in the room.

Important properties of measure NPs: **indistinguishability of referents** (see the 1D approach above), i.e., **lack of individuation**, and **no existential entailment**

4.1. Analyses

Hackl 2000: **many** is a scalar predicate, taking a degree on the quantity scale (a number) as its first argument:

Actually Hackl argues for treating **many** as an existential quantifier, but for my purposes here it is not relevant.

(19) \[ [many] = \lambda d. \lambda x. |x| = d \]

And the comparative *more than X* is treated as follows:

(20) More than five books are on the table.

\[
\begin{array}{c}
\text{er} \\
\lambda d". d"=5 \\
\lambda d' \\
\lambda d \\
\text{DP} \\
\text{VP} \\
\text{DP} \\
\text{VP} \\
\text{DP} \\
\text{VP} \\
\end{array}
\]

Informally, “more books are on the table than there are books in 5 books being on the table”

The reason for this complex structure as opposed to treating *5 books* as a degree is the infelicity of examples like ‘More than 3 people formed a square’. Hackl lodges this infelicity in the comparative phrase.

Kennedy 2015 also treats cardinals as degree-denoting, but then what do you do with μPs?

Lots of proposals (see Krifka 1990, Rothstein 2013, 2016, 2017, Rett 2015 for the context of measure nouns) treat cardinals as numbers:

Rett’s story is more complicated, actually: she seems to assume that **seven acrobats is [7 units-many] acrobats**, with a null measure function μ, but does not focus on this

(21) a. \[ [\text{liter}] = \lambda n\lambda P\lambda x[ P(x) \land \text{litr}'(x) = n ] \]
    b. \[ [\text{kilo}] = \lambda n\lambda x. \text{MEASURE}_\text{WEIGHT, KILO}(x) = n \]

Some (Grosu and Landman 1998, Scontras 2014) propose that degrees are complex entities (and cardinals are still numbers):
The syntax of modified numerals and the semantics of derived degrees (June 6-7, 2018)

(22) a. \[ \text{[three]} = \lambda P \lambda x . P(x) \land \text{DEGREE}_{P}(x) = \langle 3, P, x \rangle \] Grosu and Landman 1998

b. \[ \text{[kilo]} = \lambda k \lambda n \lambda x . \cup k(x) \land \mu_{kg}(x) = n \] Scontras 2014

Their system is incompatible with treating cardinals as degrees (infinite regress)

4.2. The polysemy of nouns

Rett 2014: NPs are ambiguous between degree and individual interpretation (also Solt 2009):

(23) a. Four pizzas are vegetarian.        individual

b. Four pizzas is more than we need.    degree

(24) [four [M-OP pizzas]] [are vegetarian]

a. \[ [M-OP pizzas] = \lambda d \lambda x . pizzas(x) \land \mu (x) = d \]

b. \[ [four M-OP pizzas] = \lambda x . pizzas(x) \land \mu (x) = 4 \]

c. \[ [four M-OP pizzas are vegetarian] = \lambda x . \text{vegetarian}(x) \land pizzas(x) \land \mu (x) = 4 \]

d. \[ = EC \exists x . \text{vegetarian}(x) \land pizzas(x) \land \mu (x) = 4 \]

For the degree reading two null operators:

(25) [four [[M-OPd M-OPe pizzas ] [ is enough ]]]

a. \[ [M-OPe pizzas] = \lambda d \lambda x . pizzas(x) \land m_{\text{quantity}}(x) = d \]

b. \[ = EC \lambda d \exists x \{pizzas(x) \land m_{\text{quantity}}(x) = d\} \]

c. \[ [ M-OPd M-OPe pizzas]] = \lambda d' . \mu_{d} (\lambda d \exists x \{pizzas(x) \land m_{\text{quantity}}(x) = d\} = d') \]

d. \[ [is enough]] = \lambda d . \text{enough}(d) \]

e. \[ [[ M-OPd M-OPe pizzas is enough]] = \lambda d' . \mu_{d} (\lambda d \exists x \{pizzas(x) \land m_{\text{quantity}}(x) = d\} = d' \land \text{enough}(d')) \]

f. \[ [four] = \lambda D . D(4) \]

g. \[ [four M-OPd M-OPe pizzas is enough]] = \mu_{d} (\lambda d \exists x \{pizzas(x) \land m_{\text{quantity}}(x) = d\} = 4 \land \text{enough}(4)) \]

This analysis crucially relies on the cardinal being external to the NP.
I also have the impression that (25) entails that 4 beers is also enough.

This doesn’t seem to work for derived measures in (16) without further stipulations

The basic intuition that an entity-denoting NP can be coerced to denote its own measure with respect to the relevant property is sound, I’m just not sure why this is done NP-externally.

The connection between regular and measure nouns is also examined for pseudo-partitives in Matushansky and Zwarts 2017 and Snyder and Barlew 2016

Snyder and Barlew 2016 basically propose that glasses is ambiguous between container and content readings, but it is also possible to derive a reflexive measure reading where glasses are filled with glasses.

5. Conclusion and further questions

The syntax of modified numerals is such that the modifier needs to be combined with the NP as a whole

This is consistent with the independently required need to construct derived degrees

Joost Zwarts and I have tried to think of a way of doing to so with 1D semantics, but it seems to lead to odd results for pseudo-partitives (cf. Snyder and Barlew 2016)
5.1. Prepositional numerals and case

Pankau 2018 assumes the structure in (1) for prepositional cardinals and shows that external case-assignment overrides case-assignment by the preposition:

(26) a. mit gegen Hundert Arbeiter-n with towards hundred worker-DAT.PL
     with approximately hundred workers
b. Peter hilftDAT umACC die zehn Männern/*Männer.
   Peter helps around the ten men.DAT/ACC
   Peter helped around ten men.

The situation might be somewhat more complex, actually. If the external preposition assigns accusative and the internal one, dative, ineffability ensues:

(27) a. *Sie gingen durchACC unterDAT 50% des Waldes.
     they walked through under 50% the.GEN forest
b. *Sie kämpften gegenACC unterDAT 50% der Angestellten.
     they fought against under 50% the.GEN employees

So it doesn’t seem to be the case that the internal preposition fails to assign case

5.2. Why are cardinals not degrees?

There's a paper (Ionin and Matushansky 2006) and a book (Ionin and Matushansky [in press]) where it is argued that cardinals should be treated as modifiers (type \(\langle e, t \rangle, \langle e, t \rangle \)).

The reason is the need to provide a system for constructing complex cardinals. Suppose that it was all wrong and cardinals refer to numbers (Frege 1884). The following issues then need to be resolved:

- How do cardinals combine with NPs?
- How do cardinals combine with measure nouns?
- How do measure nouns function without a cardinal?

Some of these questions are raised and answered in Kennedy 2015 and Rothstein 2013, 2016, 2017

Complex cardinal formation
Rothstein divides cardinals into two categories, claiming that multiplicands are not numbers. Kennedy is not concerned with complex cardinals at all

NP-internal cardinals
Rothstein assumes an \(n \rightarrow \langle e, t \rangle\) type-shift.
Kennedy hypothesizes a cardinality function \(#\) (cf. also Salmon 1997)

Cardinals in measure phrases
Rothstein takes cardinals to be arguments of measure nouns (which are relations between a number and a substance).
Kennedy is not concerned with the composition of measure phrases

Measure phrases without cardinals are generally not examined at all

6. References


Kennedy, Christopher. 2015. A “de-Fregean” semantics (and neo-Gricean pragmatics) for modified and unmodified numerals. Semantics and Pragmatics 8, pp. 1-44.


---

Ora Matushansky

The syntax of modified numerals and the semantics of derived degrees (June 6-7, 2018)


