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SOME REMARKS ON MODIFIED NUMERALS

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Barwise and Cooper (1981), Keenan (1996): modified numerals are generalized quantifiers with complex determiners:

(1) a. seventeen statues — unmodified numeral
b. [[at least ten] chains] — superlative numeral
c. [[about ten] portraits] — prepositional numeral
d. [[more than ten] visitors] — comparative numeral
e. [[exactly eleven] teachers] — precise numeral

Challenge: provide a compositional account of modified numerals that would be compatible with complex cardinals involving addition or multiplication (Ionin and Matushansky (2006)), which don't allow the bracketing in (1).

1. FOCUS ON *AT LEAST*

Existing analyses of *at least* concentrate on the role of focus, the potential referentiality of the resulting NP and the modal component.

1.1. Krifka (1999): the role of focus

Superlative and precise modified numerals involve association with focus:

(2) a. A: Three boys left.
B: No, four.

b. A: At least three boys left.
B: *No, four.

Number words can introduce alternatives without the help of focus; *at least* operates on Horn scales:

(3) $[\![\text{at least } \alpha]\!] = \text{undefined}$
$[\![\text{at least } \alpha]\!]_A = +\{X \in \text{Field}([\![\alpha]\!]_A) \mid \langle X, [\![\alpha]\!]\rangle \in [\![\alpha]\!]_A\}$

The resulting set of positively (for *at least*) or negatively (for *at most*) marked alternatives is then combined with an illocutionary operator or an intensional verb:

(4) If α is of type t, $[\![\alpha]\!]$ is undefined, and $[\![\alpha]\!]_A$ contains ±-marked alternatives, then take as new meaning $[\![\alpha]\!]$ the following: $\cup[\{p \mid +p \in [\![\alpha]\!]_A\} \cup \{\neg p \mid -p \in [\![\alpha]\!]_A\}]$
and as the new alternatives $[\![\alpha]\!]_A$ the standard alternatives (i.e., $\{[\![\alpha]\!], \langle [\![\alpha]\!], [\![\alpha]\!]\rangle\}$)

Krifka then extends his analysis to *more/less than*, *exactly* and *between.*

All these modifiers can focus on expressions other than number words:

(5) a. Mary is at least an ASSOCIATE professor (perhaps even a full professor).
b. At least [JOHN and MARY]$_F$ left.

Modified numerals do not involve a constituent consisting of the modifier and the cardinal.

There is no crucial distinction between *at least three* and *more than two*.

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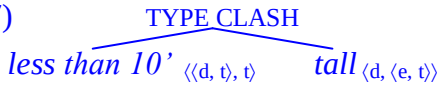
1.2. Hackl (2000): the comparative component

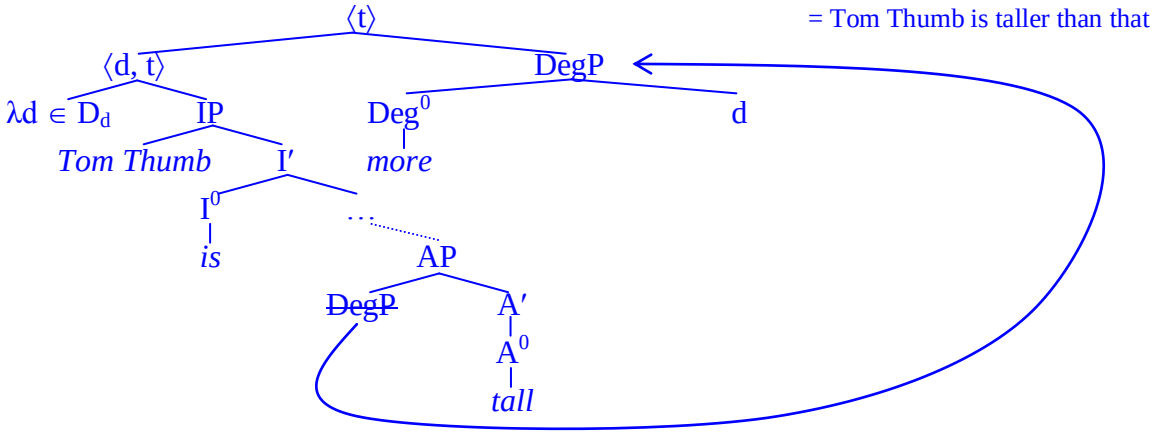
Hackl (2000): the semantics of comparative numerals in (6a-b) is analogous to the semantics of comparative constructions, (6c) (Heim (2000)): in both cases, a maximality operator applies to a degree predicate M, and cardinalities are seen as a special kind of degree.

(6) a. [[more than 10]] = λM. $\max_n$ (M(n)) > 10
b. [[fewer than 10]] = λM. $\max_n$ (M(n)) < 10
c. [[*-er* than d]] = λM. $\max_{d'}$(M(d')) > d

The comparative morpheme cannot be interpreted *in situ* and **must QR**:

(7)


Disregarding all non-essential projections, we obtain the following structure:

(8)


The comparative morpheme + comparative clause complex must raise **at least as high as the first ⟨t⟩-type node**, where λ-abstraction over degrees ensures its interpretability.

A numeral must therefore have the semantic type *d* (degrees), and be an argument of a silent quantifier *many*, which has the type ⟨d, ⟨⟨e, t⟩, ⟨⟨e, t⟩, t⟩⟩⟩:

(9) ⟦ [10 many] sandwiches⟧ = λf. $\exists$x [#x=10 & sandwiches(x) & f(x)]

The generalized quantifier over degrees *more than 10* cannot be interpreted in situ and must QR:

(10) a. Mary ate more than 10 sandwiches.
b. [[$_{DP}$ [[more than 10] many] sandwiches]]
c. [[more than 10] [λn [Mary ate [[n many] sandwiches]]]]
d. $\max_n$ ($\exists$x [#x=n & sandwiches(x) & Mary ate x]) > 10

Incorrect prediction: since comparative numerals involve QR, they should enter into scope ambiguities with other quantifiers:

(11) Every linguist read fewer than 10 abstracts.
a. $\forall$x [linguist (x) $\rightarrow$ $\max_d$ (x read d many abstracts) < 10]
b. *$\max_d$ ($\forall$x [linguist (x) $\rightarrow$ x read d many abstracts]) < 10

(11b) is true if at least one linguist read fewer than 10 articles.

Likewise, Kennedy (1997/1999): if the degree operator moves, it should be able to scope over other operators. Such readings are in fact not attested (see Heim (2000)):

(12) John is 4 feet tall. Every girl is exactly 1 inch taller than that.
a. ∀ > *-er*: ∀x [girl(x) → max {d: tall(x,d)} = 4′ + 1"]
b. **-er* > ∀: max {d: ∀x [girl(x) → tall(x,d)}] = 4′ + 1"

The reading in (12b), where the shortest girl is one inch taller than John and the rest are taller, is not available

(13) Kennedy's generalization (Heim (2000)):

If the scope of a quantificational DP contains the trace of a degree operator, it also contains that degree operator itself.

However, degree operators do interact with **intensional predicates** (Heim (2000)); therefore, degree operators can scope, as shown by the ambiguity of (14):

(14) This draft is 10 pages long. The paper is required to be exactly 5 pages longer than that.
a. *required* > *-er*: required [[exactly 5 pages -er than that][the paper be d-long]]
$\forall w \in Acc$: max {d: long $_w$ (p, d)} = 15 pages
in every possible world compatible with what is required the maximal length of the paper is exactly 5 pages more than 10 pages. In other words, the paper cannot be longer or shorter than 15 pages.

b. *-er* > *required*: [exactly 5 pages -er than that] [required [the paper be d-long]]
max {d: $\forall w \in Acc$: long $_w$ (p, d)} = 15 pages
As before, we construct the set of all degrees such that the paper is long to these degrees in any possible world compatible with what is required, and then take the maximal such degree. This maximal degree, corresponding to the length of the shortest paper compatible with what is required, has to be 5 pages more than 10 pages. Under this reading, the *minimal* length of the paper is exactly 15 pages; it can be longer than that.

Meier (2002): minimality and maximality effects with comparatives embedded in intensional contexts result from different ordering sources

Hackl (2000): comparative numerals interact with intensional predicates:

(15) John is required to read fewer than 6 books.
a. [require [[fewer than 6] [λn [John reads [[n many] books]]]]]
in every possible world compatible with what is required the maximal number of books that John reads is less than six: John shouldn't read more than 5 books.

b. [[fewer than 6] [λn [require [John reads [[n many] books]]]]]
the maximal number such that John reads that number of books in every possible world (i.e., the minimal number of books that John should read) is less than 6

Problems with Hackl's proposal:

- the complex determiner bracketing [*more than n*] is incompatible with the cross-linguistic syntax of complex cardinals
- no independent evidence for a phonologically null *many* is provided; there is no known language in which numerals occur with an overt *many*

1.3. Geurts and Nouwen (2007): scalar modifiers are not the same

Geurts and Nouwen (2007): comparative numerals are semantically and syntactically distinct from superlative numerals, although both are constructed on the basis of Horn scales that arise from focus-induced alternatives:

(16) ⟦more than α⟧ = λx∃β [β > α ∧ β(x)], where both α and β are of type ⟨e, t⟩
where > symbolizes the precedence relation on Horn scales

To use the predicate created by the comparative numeral as an argument, existential closure is required. For the downward-entailing *fewer than*/*less than* Geurts and Nouwen (2007) use **universal closure** (de Swart (2001)).

(17) a. If α is of type *t*, then⟦at least α⟧ = □α ∧ ∃β [β > α ∧ ◊β]
b. If α is of type ⟨α, t⟩, then⟦at least α⟧ = λX [□α (X) ∧ ∃β [β > α ∧ ◊β(X)]]

As a result, several differences between comparative and superlative numerals are predicted.

Comparative modifiers are defined **for predicates only**, which explains why superlative, but not comparative numerals, appear to be compatible with a **specific construal** of the numeral-containing NP:

(18) a. I will invite at most two people, namely Jack and Jill.
b. I will invite at least two people, namely Jack and Jill.
c. ? I will invite fewer than three people, namely Jack and Jill.
d. ? I will invite more than one person, namely Jack and Jill.
e. * I will invite more/fewer than two people, namely Jack and Jill.

Since referential NPs can be coerced into generalized quantifiers, but not predicates, Geurts and Nouwen (2007) correctly predict that only with superlative modifiers can the cardinal-containing NP itself get a specific construal. They also incorrectly predict that comparative modifiers should be incompatible with referential NPs.

Superlative but not comparative numerals **contain a necessity operator** which explains why the former but not the latter require the numeral-containing NP which follows the modifier to have **existential force**:

(19) a. Betty had at least four highballs.
b. Betty had more than three highballs.

(19b), and not (19a), can be used to describe a situation where Betty did not have any highballs, but instead had six margaritas, provided that, in the particular context under consideration, six margaritas is considered to outrank three highballs.

Occasionally, sentences with **comparative quantifiers are ambiguous in modal contexts**:

(20) a. You may have at most two beers. unambiguous
= it is not allowed that you have three beers

b. You may have fewer than three beers. ambiguous
= it is not allowed that you have three beers
= it is allowed that you have fewer than three beers

The universal generalized quantifier corresponding to *fewer than three beers* can outscope the modal.

The presence of the **possibility operator** in superlative numerals explains why an unmodified numeral gives rise to the **inference** in (21b), but not to that in (21c):

(21) a. Betty had three martinis.
b. Betty had more than two martinis.
c. Betty had at least three martinis.

Superlative quantifiers have a wider range of distribution (except under negation (impossible) and in DE environments (dispreferred)):

(22) a. Betty had three martinis at most/*fewer than.
b. At least/*more than, Betty had three martinis.
c. Wilma danced with at most/*fewer than every second man who asked her.

On the assumption that superlative numerals contain modal operators, they are predicted to be banned from environments where modals are disallowed.

1.4. Büring (2008): the pragmatics of superlative numerals

Büring (2008): the locus of modality on superlatives is pragmatic rather than semantic:

(23) for any set D of numbers/degrees, ⟦at least 3⟧(D) = 1 iff 3 = max(D) ∨ 3 < max(D)

The disjunction in (23) yields the modal effect:

(24) Betty drank at least three martinis.
a. The speaker is certain that Betty drank (at least) three martinis
b. The speaker is not certain that Betty drank exactly three martinis
c. The speaker is not certain that Betty drank more than three martinis.

Cummins and Katsos (2010): the disjunction inherent in ≤ / ≥ gives rise to an implicature of 'speaker uncertainty'. Similar contrasts in response patterns arise regardless of whether experimental subjects respond to the actual words *more than two* vs. *at least three*, or to the mathematical symbols > 2 vs. ≥ 3.

Superlative numerals may also be ambiguous in modal contexts: The *authoritative* reading in (25a) is incompatible with the actual requirement being, e.g., 8 characters, unlike the *speaker insecurity* reading in (25b):

(25) a. The password must be at least five characters long.
b. To become a member of this club, you must pay at least $200,000.

In unbiased contexts both readings are available:

(26) John has to read at least 3 books.
a. It has to be the case that John reads three or more books. authoritative
b. The speaker is sure that John has to read 3 books. speaker insecurity

Büring (2008): in the **authoritative** reading *at least three* scopes **below** the modal while in the **speaker insecurity** reading it scopes **above** it:

(27) a. John has to [[at least three] [λd [t_{John} read d-many books]]] authoritative
b. □ [3 = max(λd.John reads d-many books) ∨
3 < max(λd.John reads d-many books)]
'in every permitted world, the maximum number of books John reads is greater than or equal to 3'

(28) a. [at least three] [λd [John has to [t_{John} read d-many books]]] speaker insecurity
b. 3 = max(λd. □[John reads d-many books]) ∨
3 < max(λd. □ [John reads d-many books])
'the highest number *n* s.t. in every permitted world, John reads that n (or more) books is greater than or equal to three'

In order to derive the two scopal possibilities *at least 3* must be a phrasal constituent (an XP).

Island-sensitivity is also incorrectly predicted:

(29) They must hire three assistant professors and at least one associate professor.
  a. authoritative reading: the requirement is to hire at least one associate professor
  b. speaker insecurity reading: I don't know how many associate professors they have to hire, but it's no fewer than one

Büring (2009): *at most* necessitates the third construal:

(30) a. He had to wait at most three months. speaker insecurity reading
  b. They could stay underwater for at most 10 minutes.

(31) a. The meat you use must have at most 5% of fat. authoritative reading
  b. The meat you use may have at most 5% of fat.

(31b) cannot be achieved by scoping *at most 5%* either above or below the modal: instead the numeral alone must scope above the modal.

1.5. Beck (2009): focus semantics for degree operators

Oda (2008): *exactly*-differentials are themselves degree operators and can scope:

(32) ⟦[exactly five pages]⟧ = λD . max (D) = 5 pages

The ambiguity exhibited by positive comparatives under modals is not due to the movement of the comparative morpheme, but to the QR of the differential itself, with the wide scope of the *exactly*-differential giving rise to the minimal difference reading:

(33) max {d': ∀w ∈ Acc [max {d: the paper is d-long in w]} ≥ 10pp+d'} = 5 pages
The largest degree d' such that the paper is d'-much longer than 10 pages in every relevant world is 5 pages, i.e., the **minimal difference** to 10 pages required is 5 pages

Beck (2009): NPs containing *exactly* exhibit scope ambiguity with modals but not with other quantifiers:

(34) You are allowed to write exactly 5 pages.
  a. allowed [[exactly 5 pages] [λd . [you write d-much]]]
  ∃w ∈ Acc [max {d: you write d-much} = 5 pages]
  It is permitted for you to write exactly 5 pages (but you can write more than that, too)
  b. [[exactly 5 pages] [λd . allowed [you write d-much]]]
  max {d: ∃w ∈ Acc [you write d-much]} = 5 pages
  The maximum you are allowed to write is exactly 5 pages

(35) You are required to write exactly 5 pages.
  a. required [[exactly 5 pages] [λd . [you write d-much]]]
  ∀w ∈ Acc [max {d: you write d-much} = 5 pages]
  It is required that you write exactly 5 pages
  b. [[exactly 5 pages] [λd . required [you write d-much]]]
  max {d: ∀w ∈ Acc [you write d-much]} = 5 pages
  The minimum you are required to write is exactly 5 pages

To explain the Heim-Kennedy generalization Beck invokes focus: intervention effects occur when an alternative semantics (Rooth (1985)) is involved (Beck (2006, 2007)):

(36) a. The paper has to be exactly 10 pages long.
  b. [EXACT D [~D [$_{XP}$ have to [the paper be *exactly* [10 pages]$_F$ long]]]]

The adverb *exactly* introduces alternatives but is not itself interpreted. These alternatives are evaluated by the silent operator EXACT (Krifka's ASSERT), whose meaning is that of *only*, but with all components at the level of assertion:

(37) $[\![\text{EXACT}]\!]\ (D)\ (p) = 1$ if and only if $p\ \&\ \forall q \in D\ [\neg\ (p \rightarrow q) \rightarrow \neg\ q]$

The ordinary value of (36a) has no *exactly*; its alternative semantic value involves all possible alternatives to the cardinal in focus:

(38) a. $[\![XP]\!]_O = \forall w \in Acc$ [the paper is 10 pages long in w]
b. $[\![XP]\!]_{Alt} = \{\forall w \in Acc$ [the paper is n pages long in w]: $n \in N\}$

The alternative-evaluating EXACT operator may be merged higher or lower than the modal:

(39) $\forall q \in [\![XP]\!]_{Alt}\ [\neg\ ([\![XP]\!]_O \rightarrow q) \rightarrow \neg\ q]$ if and only if
$\forall n \in N: n>10 \rightarrow \neg\ [\forall w \in Acc$ [the paper is n pages long in w]]
The most informative true proposition of the form 'The paper has to be n pages long' is 'The paper has to be 10 pages long'

The alternative-evaluating EXACT operator cannot outscope nominal or adverbial quantifiers because those, unlike modals, have been suggested to come with their own ~ operator.

Heim (2000): *less*-comparatives also show scope ambiguities under modals:

(40) [This draft is 10 pages long.] The paper is required to be less long than that.
a. The minimum length required for the paper is less than 10 pages. less > required
b. Papers longer than 10 pages are not allowed. required > less

Beck (2009): the negative degree morpheme *little* (cf. Heim (2006)) also introduces focus alternatives, which are evaluated by the higher covert operator AT MOST. It is interpreted as *much*.

Problems:

- No connection between the modifier (*exactly*, *little*) and the alternative-evaluating operator predicts a mix-and-match generation
- Island violations and non-local configurations are predicted
- The availability of *precisely*, alongside *exactly*, is not expected, nor is the lexical nature of such modifiers
- Beck (2009): the behavior of comparatives in ellipsis and ACD (Heim (2000)) is not predicted

A link between comparatives and numerals is reaffirmed.

1.6. Kennedy (2010): the scope of numerals

Kennedy (2010), see also Rothstein (2011): number words are degrees

Carston (1998): cardinal-containing NPs in the context of a modal give rise to minimum and maximum readings in the absence of a comparative:

(41) Mary is required to publish exactly two papers to get tenure.
a. exact requirement: Mary must publish two papers and no more
b. minimal requirement: Mary cannot publish fewer than two papers.

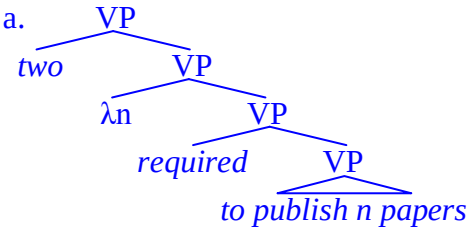
Beck (2009): *exactly* introduces focus alternatives that are evaluated by an operator that can outscope the modal.

Kennedy (2010): number words are scope-taking degree quantifiers:

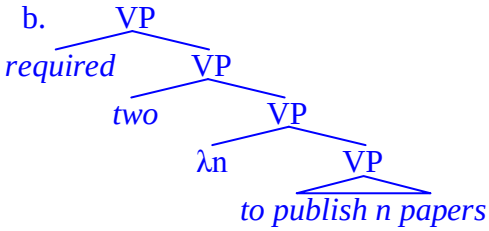
(42) $[\![\text{seven}]\!] = \lambda P \in D_{\langle d, t\rangle}$. max $\{m : P(m)\} = 7$

The two interpretations of (41) therefore arise as a result of the different scope positions of the number word:

(43) a.



b.



Unlike in Rothstein's approach, the number word cannot be interpreted *in situ*, since its sister, the NP, is generally not assumed to have the semantic type ⟨d, t⟩.

Problem: though referential DPs and coordinate structures are islands for QR (Ruys (1992)), they can nonetheless contain number words:

(44) a. those two books
b. the eight planets of the Solar System
c. two specific girls

Needless to say, cardinals are incorrectly predicted to be sensitive to other islands as well.

Unattested interpretations are predicted with number words scoping higher then quantifiers:

(45) a. No one read five books.
b. max {n : ¬∃x [x read n books]} = 5

Predictions for modified numerals are unclear.

2. WHAT CAN BE DONE ABOUT MODALITY?

It is easy to show that the minimal and maximal readings of modal content are available in the absence of comparatives or number words (the (a) examples come from Kennedy (2010); see also Büring (2008)):

(46) obligation + *at least*:
a. In Britain, you have to be 18 to drive a car.
b. In the Middle Ages, you had to own property/be a landowner to really matter.

(47) obligation + *exactly* (*at least* + *at most*?):
a. In "Go Fish", each player has to start with seven cards.
b. In another game, each player has to start with a king, a deuce and an ace.

(48) permission + *at least*:
a. Kim can withdraw $1000 at once. Büring (2008)
b. [I've had a raise] I can buy a new car now/I can relax now.

(49) permission + *at most*:
a. She can have 2000 calories without putting on weight.
b. If you visit our bar between 5 and 7 PM, you can get a free drink.

Heim's ambiguity may therefore not result from the movement of the comparative morpheme, as may the ambiguity exhibited by modified numerals.

Where do the two readings of modal content come from?

One straightforward option: **existential quantification over eventualities**.

All existentially quantified NPs are compatible with both an exact and a minimal readings:

(50) a. Julian HAS bought a house. In fact, he has bought two. minimal
b. You said I would win if I received three A's. Well, I've got four!

(51) a. Jennifer was writing a letter when the doorbell rang. exact
b. Robert left after winning three games.

Since modal content must involve an eventuality variable, the minimal reading of the modal content is expected. The exact reading has to be derived (just like with existentials).

Büring (2008): evidence for movement:

(52) a. You need to fill out at least 50 forms for that. authoritative/speaker insecurity
b. It is required that you fill out at least 50 forms for that. authoritative only

Alternative explanation: a full-fledged matrix clause introduces its own eventuality variable.

(53) obligation + *at least*
a. The password must be at least five characters long.
b. To become a member of this club, you have to pay at least $200,000.

- The **authoritative reading** (analyzed by Büring as the low scope of *at least* with respect to the modal) corresponds to the *exactly* reading of the modal content: the speaker knows precisely what the requirement is.
- The **speaker insecurity reading** (Büring's high position of *at least* with respect to the modal) corresponds to the minimal reading of the modal content.

We now predict a two-way ambiguity for every sentence containing a scalar modifier in the scope of a modal:

(54) a. The password must be more than five characters long.
b. To become a member of this club, you have to pay more than $200,000.

However:

(55) To become a member of this club, you have to pay **no less than $200,000**.
a. authoritative reading: the requirement is that you pay $200,000 or more
b. speaker insecurity reading: I don't know how much you have to pay, but it is at least $200,000

Büring's ambiguity is not due to the disjunctive interpretation of *at least*.

Büring (2009): disjunction under possibility modals doesn't trigger the implicatures necessary to activate the authoritative reading. However, authoritative readings with possibility modals do exist:

(56) a. This boat can sleep at least six people.
b. #You are allowed to eat at least two candy bars.

Büring (2009), following Klinedinst (2007): existential modals involve a hidden universal quantification

No special assumptions necessary on our story.

However, *at most* is more tricky:

(57) a. He had to wait at most three months. speaker insecurity reading
b. They could stay underwater for at most 10 minutes.

(58) a. The meat you use must have at most 5% of fat. authoritative reading
b. The meat you use may have at most 5% of fat.

This is not a special feature of *at most*:

(59) a. You may withdraw **no more than/up to** $1000 a day.
b. You may withdraw $1000 **tops/maximum** a day.

(60) permission + *at most*:
a. She can have 2000 calories without putting on weight.
b. If you visit our bar between 5 and 7 PM, you can get a free drink.

I'm not sure how to get this interpretation of the modal, which seems to assert something like "and nothing else is allowed". Scoping the cardinal (cf. Kennedy (2010)) seems too drastic a solution, which is besides incompatible with the cross-linguistic syntax of cardinals (cf. Ionin and Matushansky (2006)).

Hypothesis: it has something to do with focus:

(61) -- What can you do in prison?
-- You can wait to be released.

Something like: the maximally informative true proposition of the form "You can X" is...

To be continued...

3. MORE THAN ONE *MORE THAN ONE*

Ambiguity of the English *more*:
- *more* as the synthetic comparative of *much*
- *more* as the freestanding form of the comparative affix *-er*, '*mo*-support' (Bresnan (1973); Corver (1997))

(62) a. Much is good, more is better. [synthetic comparative of *much*]
b. *mo*-re intelligent : smart-er [*mo*-support]

Two types of comparatives (Hankamer (1973), Napoli (1983), Heim (1985), Lechner (1998, 2001), Pancheva (2006))
- clausal: *than* combines with a full or partially elided finite CP
- phrasal: *than* combines with a constituent smaller than a CP

(63) a. Mary is taller than John is. [clausal comparative]
b. Mary is taller than John / than 5 feet. [phrasal comparative]

Matushansky and Ionin (to appear): cardinal-containing comparatives like *more/fewer than five children* can encode any of the four resulting options, but this difference is not about bracketing: we analyze all four options as having the bracketing in (64b), not (64a):

(64) a. [[more than five] sandwiches]
b. [more than [five sandwiches]]

Empirical evidence for having four different options: in Russian, *more than five children* can be translated in four different ways:
- phrasal comparative (*more* + Genitive-marked NP) vs. clausal comparative (*more* + wh-expression) (cf. Heim (1985), Lechner (1998, 2001), Pancheva (2006))
- two *more's*: *bol'še* (suppletive comparative of *mnogo*, 'much/many') vs. *bolee* (*mo*-support)

(65) a. bol'še pjati detej [suppletive comparative of *mnogo*; phrasal]
more five-GEN child-GEN.PL
b. bol'še čem pjat' detej [suppletive comparative of *mnogo*; clausal]
more than five-NOM=ACC child-GEN.PL
c. bolee pjati detej [*mo*-support; phrasal]
more five-GEN child-GEN.PL
d. bolee čem pjat' detej [*mo*-support; clausal]
more than five-NOM=ACC child-GEN.PL

NB: The same pattern is observed with *fewer than five children*: *men'še* is the suppletive comparative of *malo* 'a little', while *menee* is *less*.

3.1. *Many* vs. *much* readings

English cardinal-containing comparatives are ambiguous between *many* and *much* readings:

(66) more than five sandwiches
  a. '*many* reading': ≈ *six or more sandwiches*
  b. '*much* reading': ≈ *something more substantial than five sandwiches*

(67) I ate more than five sandwiches…
  a. '*many* reading': …I ate six!
  b. '*much* reading': …I ate five sandwiches plus a bowl of soup!
  c. '*much* reading': …I ate a whole bowl of soup!

The same effect is seen with measure nouns:

(68) I bought more than a pound of apples…
  a. '*many* reading': …I bought a pound and a half.
  b. '*much* reading': …I also bought some bananas.
  c. '*much* reading': …I bought two containers of strawberries.

The *much* vs. *many* reading correlates with the degree adjective of equative constructions:

(69) a. I ate as much as five sandwiches – in fact, I ate six pastries.
  b. I ate as many as five sandwiches – #in fact, I ate six pastries.

In Russian, the *many* reading is available to all four comparative types, but the *much* reading is available only to clausal *bol'še* comparatives:

(70) a. Ja s'jela bol'še čem pjat' buterbrodov.
  I ate more than five-ACC sandwiches.
  '*many* reading': ≈ *six or more sandwiches*
  '*much* reading': ≈ *something more substantial than five sandwiches*

  b. Ja s'jela bolee čem pjat' buterbrodov.
  I ate more than five-ACC sandwiches
  '*many* reading': ≈ *six or more sandwiches*
  #'*much* reading': ≈ *something more substantial than five sandwiches*

  c. Ja s'jela bol'še/bolee pjati buterbrodov.
  I ate more five-GEN sandwiches
  '*many* reading': ≈ *six or more sandwiches*
  #'*much* reading': ≈ *something more substantial than five sandwiches*

There's cross-linguistic difference with respect to the readings of phrasal comparatives.

3.2. Referentiality

The above examples show that comparatives in English and Russian are compatible with cardinal-containing NPs, which we analyze as having the semantic type of predicates (⟨e, t⟩) (Landman (2003); Ionin and Matushansky (2006); Geurts and Nouwen (2007)).

However, comparatives are also compatible with referential (type *e*) expressions:

(71) a. I invited more than (just) Peter and Mary.
  b. I read more than these five books.

Note that comparatives over referential expressions have only the *much* reading:

(72) '*much* reading':
a. I invited more than Peter and Mary – I also invited their mother.
b. I read more than these five books – I also read an encyclopedia.

(73) '*many* reading':
a. I invited more than Peter and Mary – #I invited three people.
b. I read more than these five books – #I read six books.

NB: Comparatives do not seem to be compatible with true quantified expressions (type ⟨⟨e, t⟩, t⟩): **I invited less than everyone; *More than anyone came;* etc. Evidence that the cardinal-containing NP inside a comparative is not existentially quantified comes from the fact that *I read fewer than five books* does not entail the existence of five books.

In Russian, only the comparative type that is compatible with the *much* reading (the clausal comparative with *bol'še*) is compatible with referential expressions:

(74) a. Ja priglasila bol'še/*bolee čem Petju i Mašu.
I invited more than Peter-ACC and Mary-ACC
'I invited more than Peter and Mary.'

b. Ja pročitala bol'še/*bolee čem èti pjat' knig.
I read more than these five-ACC book-GEN.PL
'I read more than these five books.'

c. *Ja priglasila bol'še/bolee Peti i Maši.
I invited more Peter-GEN and Mary-GEN

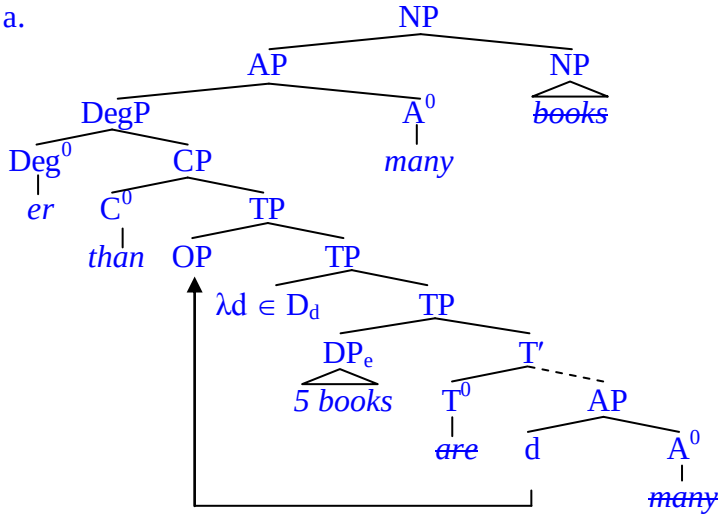
d. *Ja pročitala bol'še/bolee ètix pjati knig.
I read more these five-GEN book-GEN.PL

3.3. Clausal comparative numerals

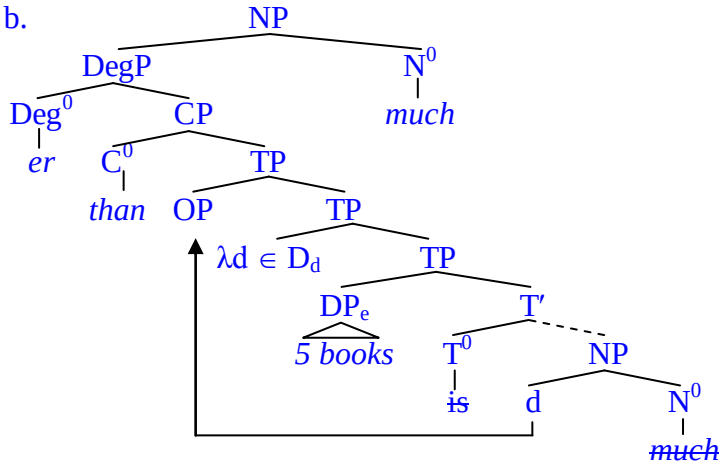
Matushansky and Ionin (to appear): clausal comparatives are full CP structures with either an underlying many or an underlying *much*, thus deriving the two readings of (68b).

(75) clausal comparative numerals:

a. *many*-reading



b. *much*-reading

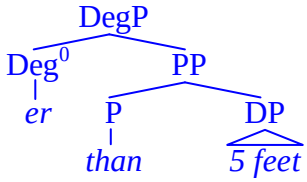


The fact that *five books* in (75a,b) is a regular subject and therefore can have type *e* or type ⟨⟨e, t⟩, t⟩ accounts for the availability of referential expressions in clausal comparatives.

3.4. Phrasal comparative numerals

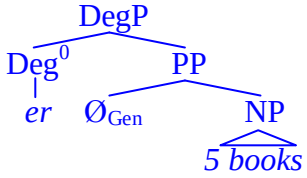
Pancheva (2006): comparatives such as *taller than five feet* are analyzed as degrees:

(76) [Pancheva (2006)]



Applying the degree analysis to Russian cardinal-containing phrasal comparatives with *bolee*:

(77) cardinal-containing phrasal comparative with *bolee:* degree analysis



Hypothesis: An NP that denotes in the count domain can be converted into a degree:
NB: This is similar to analyses of degree relatives (Carlson (1977), Heim (1987), Grosu and Landman (1988)).

(78) $P_{\langle e,t\rangle} \rightarrow \iota d$ s.t. $\forall x\, [P(x) \rightarrow d = \max \{d' : Q(d',x)\}$ where Q is contextually provided

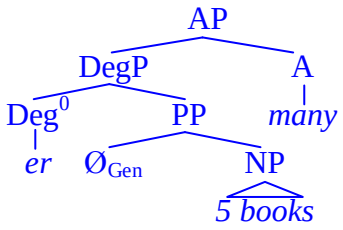
In other words, for an NP like *five books*, we obtain the degree such that it is the projection of any five-book individual onto the contextually provided scale.

Supporting evidence: any cardinal-containing NP can be used as a measure phrase:

(79) a. The series is five books long.
b. The wall is five windows wide.

Turning to *bol'še*, we assume the same analysis as for (79), with *many* replacing *long/wide*:

(80) cardinal-containing phrasal comparative with *bol'še:* degree analysis



The degree analysis of phrasal comparatives explains their incompatibility with referential expressions: a referential expression cannot become a degree.

(74) c. *Ja priglasila bol'še/bolee Peti i Maši.
I invited more Peter-GEN and Mary-GEN

d. *Ja pročitala bol'še/bolee ètix pjati knig.
I read more these five-GEN book-GEN.PL

Treating comparative numerals in the terms of comparatives predicts that they should behave as comparatives (cf. Hackl (2000)).

A potential problem for our analysis of clausal comparatives: the clausal comparative appears to be transparent for case assignment:

(81) a. My prišli s (bol'še/bolee čem) pjat'ju studentami.
we came with more than five-INSTR student-INSTR.PL
'We came with (more than) five students.'

b. My podarili podarki (bol'še/bolee čem) pjati druz'jam.
we gave gifts more than five-DAT friend-DAT.PL
'We gave gifts to (more than) five friends.'

This is not a problem for the analysis of cardinal-containing comparatives proposed by Hackl (2000), for whom *More than five books are on the table* means, informally "More books are on the table than there are books in books being on the table".

4. SUMMARY

Ambiguities exhibited by modified numerals and comparatives under modals are due to the modals rather than to the scalar modifiers or comparatives.

The modal interpretation of superlative numerals results from the disjunction in their meaning (Büring (2008)).

The differences in the distribution of comparative and superlative modified numerals reflect both their different syntax (adjunction vs. complementation) and their different semantics.

5. BIBLIOGRAPHY

Barwise, Jon and Robin Cooper (1981). Generalized quantifiers and natural language. *Linguistics and Philosophy* 4, pp. 159-219.

Beck, Sigrid (2006). Intervention effects follow from focus interpretation. *Natural Language Semantics* 14/1, pp. 1-56.

Beck, Sigrid (2007). The grammar of focus evaluation. In U. Sauerland and H.-M. Gärtner, eds., *'Interfaces + Recursion = Language'? Chomsky's Minimalism and the View from Syntax and Semantics*. Berlin: Mouton de Gruyter, pp. 255-280.

Beck, Sigrid (2009). DegP scope reanalyzed. Ms., University of Tübingen.

Bresnan, Joan (1973). Syntax of the comparative clause construction in English. *Linguistic Inquiry* 4, pp. 275-343.
Büring, Daniel (2008). The least *at least* can do. In C. B. Chang and H. J. Haynie, eds., *Proceedings of the 26th West Coast Conference on Formal Linguistics*. Somerville, Massachusetts: Cascadilla Proceedings Project, pp. 114-120.
Büring, Daniel (2009). *At least* and *at most*: the logic of bounds and insecurity. Paper presented at *MIT colloquium*, April 24, 2009, MIT.
Carlson, Gregory Norman (1977). Amount relatives. *Language* 53/3, pp. 520-542.
Carston, Robyn (1998). Informativeness, relevance and scalar implicature. In R. Carston and S. Uchida, eds., *Relevance theory: Applications and implications*. Amsterdam: John Benjamins, pp. 179-236.
Corver, Norbert (1997). *Much*-support as a last resort. *Linguistic Inquiry* 28/1, pp. 119-164.
Cummins, Chris and Napoleon Katsos (2010). Comparative and superlative quantifies: Pragmatic effects of comparison type. *Journal of Semantics* 27, pp. 271-305.
Geurts, Bart and Rick Nouwen (2007). At least *et al*.: The semantics of scalar modifiers. *Language* 83, pp. 533-559.
Grosu, Alexander and Fred Landman (1988). Strange relatives of the third kind. *Natural Language Semantics* 6, pp. 125-170.
Hackl, Martin (2000). Comparative quantifiers. Ph.D. thesis, MIT.
Hankamer, Jorge (1973). Why there are two *than*'s in English. In C. Corum, T. C. Smith-Stark and A. Weiser, eds., *Papers from the 9th regional meeting of the Chicago Linguistics Society (CLS)*. Chicago: Chicago Linguistics Society, pp. 179-191.
Heim, Irene (1985). Notes on comparatives and related matters. Ms., University of Texas, Austin.
Heim, Irene (1987). Where does the definiteness restriction apply? Evidence from the definiteness of variables. In E. Reuland and A. Ter Meulen, eds., *The representation of (in)definiteness*. Cambridge, MA: MIT Press, pp. 21-42.
Heim, Irene (2000). Degree operators and scope. In B. Jackson and T. Matthews, eds., *Proceedings of Semantics and Linguistic Theory (SALT) 10*. Ithaca, New York: CLC Publications, Department of Linguistics, Cornell University, pp. 40-64.
Heim, Irene (2006). Little. In C. Tancredi, M. Kanazawa, I. Imani and K. Kusumoto, eds., *Proceedings of Semantics and Linguistic Theory (SALT) 16*. Ithaca, New York: CLC Publications, Department of Linguistics, Cornell University.
Ionin, Tania and Ora Matushansky (2006). The composition of complex cardinals. *Journal of Semantics* 23/4, pp. 315-360.
Keenan, Edward L. (1996). The semantics of determiners. In S. Lappin, ed., *The Handbook of Contemporary Semantic Theory*. Oxford: Blackwell, pp. 41-64.
Kennedy, Christopher (1997/1999). Projecting the adjective. The syntax and semantics of gradability and comparison. Ph.D. thesis, University of California, Santa Cruz. New York: Garland.
Kennedy, Christopher (2010). The number of meanings of English number words. Paper presented at *University of Illinois*, September 16, 2010.
Klinedinst, Nathan (2007). Plurality and Possibility. Ph.D. thesis, UCLA.
Krifka, Manfred (1999). At least some determiners aren't determiners. In K. Turner, ed., *The semantics/pragmatics interface from different points of view*. Current Research in the Semantics/Pragmatics Interface: Elsevier Science, pp. 257-291.
Landman, Fred (2003). Predicate-argument mismatches and the adjectival theory of indefinites. In M. Coene and Y. D'hulst, eds., *The syntax and semantics of noun phrases*. Linguistics Today. Amsterdam and Philadelphia: John Benjamins, pp. 211-237.
Lechner, Winfried (1998). Comparatives and DP-structure. Ph.D. thesis, University of Massachusetts, Amherst.
Lechner, Winfried (2001). Reduced and phrasal comparatives. *Natural Language & Linguistic Theory* 19, pp. 683-735.
Matushansky, Ora and Tania Ionin (to appear). More than one solution. *Proceedings of CLS 47*. Chicago: Chicago Linguistic Society, University of Chicago.

Meier, Cécile (2002). Minimality and maximality in comparatives. In G. Katz, S. Reinhard and P. Reuter, eds., *Sinn and Bedeutung VI: Proceedings of the 6th Annual Meeting of the Gesellschaft für Semantik*. Osnabrück: Publications of the Institute of Cognitive Science, University of Osnabrück, pp. 275-287.

Napoli, Donna Jo (1983). Comparative ellipsis: A phrase structure account. *Linguistic Inquiry* 14/4, pp. 675-694.

Oda, Toshiko (2008). Degree Constructions in Japanese. Ph.D. thesis, University of Connecticut, Storrs.

Pancheva, Roumyana (2006). Phrasal and clausal comparatives in Slavic. In J. Lavine, S. Franks, M. Tasseva-Kurktchieva and H. Filip, eds., *Proceedings of FASL 14: The Princeton Meeting*. Ann Arbor, Michigan: Michigan Slavic Publications.

Rooth, Mats (1985). Association with focus. Ph.D. thesis, University of Massachusetts, Amherst.

Rothstein, Susan (2011). Numbers: counting, measuring and classifying. Paper presented at *Sinn und Bedeutung 16*, September 6-9, 2011, Utrecht University.

Ruys, E.G. (1992). The Scope of Indefinites. Ph.D. thesis, Utrecht University.

de Swart, Henriette (2001). Weak readings of indefinites: type-shifting and closure. *The Linguistic Review* 18/1, pp. 69-96.

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