1. **INTRODUCTION: PREPOSITIONAL MEASURE PHRASES**

Starting point: the syntax and compositional semantics of examples like (1):

(1) a. Don't touch the steering wheel if you have drunk over five glasses of wine.
    b. I ate around a pound of jam.
    c. The mass of the meteorite was estimated at under 66 tons.
    d. I was swimming between a kilometer and a mile four days a week.

Issues to be discussed and hopefully resolved:

- **internal semantics:** the normal semantics of spatial prepositions requires an entity-denoting internal argument. However, e is not the usually assumed denotation for measure phrases, especially if they do not combine with a substance NP (1c,d)

- **external syntax and semantics:** the output of spatial PPs is generally not taken to be an entity and a PP is generally not a good substitute for an argument NP

The core of the proposal: measure nouns denote abstract containers located in a vertically oriented half-open one-dimensional space. Consequences:

- measure phrases are entities and can therefore combine with prepositions
- **vertical orientation** follows from the container concept
- **algebra of scalar addition and multiplication**, i.e., the scalar structure, follows from the properties of one-dimensional space
- the interpretation of spatial prepositions is unchanged
- constraints on the choice of prepositions are explained
- unified approach to abstract and concrete pseudo-partitives becomes possible
- lack of other dimensions makes it possible to account for the container-content ambiguity noted for pseudo-partitives (Selkirk 1977, Landman 2004, Grimshaw 2007, Rothstein 2009a, Partee and Borschev 2012, Duek and Brasoveanu 2015, etc.).

The bigger picture: reconstructing degrees as entities in concrete 1D space without the need to postulate a special semantic type or sort

2. **THE SPECIFICS OF THE PROBLEM**

Many different approaches to the semantics of spatial prepositions (Wunderlich 1991, Zwarts and Winter 2000, Kracht 2002, Bateman, Hois, Ross and Tenbrink 2010, etc.)

2.1. **The syntax of the preposition**

To the best of our knowledge, **almost no prior work** on measure uses of spatial prepositions

The semantics and pragmatics of **up to** (Nouwen 2008, 2010, Schwarz, Buccola and Hamilton 2012, Blok 2013, 2016, [to appear]): connection to the directional preposition exploited, but not derived

The syntax and semantics of prepositional numerals (Plank 2004, Corver and Zwarts 2006):

(2) Ik reken op [rond de twintig kinderen].
    Dutch
    I count on round the twenty children
    *I count on approximately twenty children.*

Corver and Zwarts 2006: structural ambiguity:

(3) a. [around two] books
    b. around [two books]
Problem for this approach: prepositional measures without a cardinal:

(4)  
  a.  around a pound  
  b.  between a kilometer and a mile

Potential extension: structural ambiguity with prepositional pseudo-partitives -- only on the assumption that the cardinal and the measure noun form a constituent:

(5)  
  a.  PP  
      \[ \text{under} \quad \text{Num} \quad \text{NP} \]  
      \[ \text{of} \quad \text{water} \]  
      \[ \text{three} \quad \text{liters} \]  
  b.  PP  
      \[ \text{of} \quad \text{water} \]  
      \[ \text{three} \quad \text{liters} \]  

Claim: assuming structural ambiguity does not solve anything

- doesn't help with prepositional measures, like (4)
- or with the measure PPs without a measure noun (about twenty books)
- still requires the explanation of how prepositions combine with measure NPs
- and with cardinals
- predicts additional structural ambiguity for PP measures containing a cardinal

Plank 2004 for the syntax of the preposition

2.2. The syntax of pseudo-partitives


(6)  
      PP  
      \[ \text{up to/about...} \quad \text{Num} \quad \text{NP}_1 \]  
      \[ \text{of} \quad \text{water} \]  
      \[ \text{three} \quad \text{liters} \]  

NB: the label Num is used for expository purposes only; no positive commitment to the mode of combining a cardinal with its sister is required at this point. We will assume the cascade structure of Ionin and Matushansky 2006

NB: Semantically the simplest structure is with the substance PP merging higher than the approximative P -- syntactically highly unlikely

Measure noun as the head: NP-internal agreement (Ruys [to appear], cf. van Gestel 1986):

(7)  
  a.  die \quad \text{éne} \quad \text{liter} \quad \text{water}  
      \[ \text{that.C one liter.C water.N} \]  
      \[ \text{that one liter of water} \]
b. het onsje cocaïne  
the.N metric.ounce.DIM.N cocaine.C  
the ounce of cocaine

Head-complement relation: visible **construct state morphology** for container nouns:

(8) šloša bakbukey yayin  
three bottles.CS wine  
three bottles of wine  

Hebrew, Rothstein 2011a

However, no visible construct state morphology with measure nouns; assuming the same structure, further stipulations are needed one way or the other.

Also: case-assignment to and inside the pseudo-partitive, c-selection, semantic composition (see appendix)

### 2.3. The issue of locative semantics

The problem of prepositional measures arises with any constituency and labeling: **How does a spatial preposition combine with a measure NP to yield a non-spatial reading?**

And of course, the substance NP does not have to be present, nor does the numeral:

(9) a. The temperature is below 20°F.  
b. We drank over a liter of vodka.

For the view that treats measure phrases as denoting in a separate domain (degrees) changing the meaning of the relevant prepositions is required

Core intuition: the **locative metaphor** (Lakoff and Johnson 1980, Lakoff and Núñez 2000): degrees can be metaphorically interpreted as positions on the vertical scale (see Plank 2004, Nouwen 2016 for numerals)

**Formalization is still required for the mapping** between the normal denotation of measure phrases and their metaphorical interpretation

Our proposal: **there’s no mapping** – measures denote in the same domain as other sortals except for being 1D

### 3. **Measure nouns as abstract containers**

(10) a. The picture is **over** the mantel.  
*over* expresses a vertical relation between two material objects in 3D space  

b. I ate **over** a pound of jam.  
*over* expresses a vertical relation between two abstract containers in 1D space
3.1. **The concept of an abstract container**

Stereotypical properties of *concrete* containers:
- **verticality**: a container must be vertical to contain substances
- **measurement**: a container can map different substances to the same volume unit

Natural properties of *abstract* containers:
- conceptualized as one-dimensional
- no distinction between container and content (due to one-dimensionality)
- generalized to all quantities (weight, length, ...)
- share one natural zero point (the "ground"; cf. Nouwen 2016), differ in height
- abstract containers can be stacked on top of each other
- two abstract containers with the same height and content are indistinguishable unless stacked

3.2. **Available spatial building blocks** (simplified; see Appendix I)

Spatial building blocks in vector-space semantics (Zwarts and Winter 2000)

Two types in addition to $e$ and $t$:
- type $p$ of points
- type $v$ of vectors, represent relations between points

Functions in spatial semantics:
- $\text{LOC}$ maps an entity to its spatial boundary* (type $\langle p, t \rangle$)
- a preposition maps a boundary to a particular set of vectors (type $\langle v, t \rangle$)
- $\text{LOC}^-$ maps a set of vectors to the set of entities (type $\langle e, t \rangle$) that are located at those vectors

(* the only adjustment necessary to the vector space semantics, notational variant for 3D, but crucial for 1D, part of the general schematization/idealization operations in spatial language, cf. Herskovits 1986)

![Diagram](image)

$$x \quad \text{LOC}(x) \quad \text{OVER}(\text{LOC}(x)) \quad \text{LOC}^-(\text{OVER}(\text{LOC}(x)))$$

3.3. **Working with containers** (simplified; see Appendix)

Two necessary functions in container semantics:
- $\text{CONT}$ uses objects as containers filled with a substance (kind)
  - e.g., $\text{CONT}(\text{jar})(\text{jam}) =$ set of jars filled with jam
  - e.g., $\text{CONT}(\text{pound})(\text{jam}) =$ set of pounds filled with jam
- $\text{CONT}^-$ maps full containers to their contents
  - e.g., $\text{CONT}^- (\text{CONT}(\text{jar})(\text{jam})) =$ set of jam portions in jars
  - e.g., $\text{CONT}^- (\text{CONT}(\text{pound})(\text{jam})) =$ set of jam portions of a pound

Abstract containers in 1D space:
- for every container $c$, $\text{LOC}(c)$ is the singleton boundary containing only the top point (because the bottom point is always zero and therefore irrelevant)
for every region of vectors $R$, $\text{LOC}^{-1}(R)$ returns the set of containers of which the top coincides with a vector’s endpoint

- stacking of containers $\sim$ vector addition of their boundaries, multiple stacking $\sim$ scalar multiplication
- one dimension (vertical), one substance, one origin, obligatory grounding (the bottom of an object is either zero or the top of another)

### 3.4. The compositional structure

Of the phrase *over a pound of jam* with the meaning ‘the set of portions of jam whose volume exceeds one pound’:

$\langle e, t \rangle$

the set of contents of these containers

$\text{CONT}^{-1}(e, t)$

the set of containers whose top is given by these vectors

$\langle v, t \rangle$

the set of vectors whose endpoint is higher than that top

$\text{LOC}^{-1}(v, t)$

the set of points corresponding to the top of that container

$\langle p, t \rangle$

one pound container filled with jam

$\text{LOC}^{-1}(p, t)$

the set of pound containers filled with jam

$\langle k, (e, t) \rangle$

the kind of jam

Remarks about $\text{CONT}$ and $\text{CONT}^{-1}$:

- the core meaning of measure nouns may be transitive, in which case $\text{CONT}$ is not necessary
- a structural representation of $\text{LOC}$, $\text{CONT}$ and their reversals is not necessary, but $\text{CONT}^{-1}$ may have a structural syntactic representation corresponding to a conversion from a PP to an NP (i.e., as a nominalizer):

(12) a. **the [over 9 million liters of water and 50,000 filters distributed by FEMA]**

b. **for the duration of those up to ten minutes**

---

The meaning of phrases with intransitive measure nouns and numerals can be derived with the assumptions of 3.2 and 3.3:

(a) under five liters
(b) \([\text{LOC} \leftarrow \text{LOC} \left[ \text{LOC} \left[ \text{INDEF} \left[ \text{five liters} \right] \right] \right] \] \)
(c) ‘the set of volume containers of which the top coincides with the endpoint of a vector that points downward from the top of a stack of five one-liter containers’

4. **Summary**

Core stipulation: the existence of abstract spaces with reduced structure

Independent evidence: spatial prepositions with result predicates (*change from a prince into a frog*) and result states (*loving me to death/into an early grave*)

Consequences:

- the standard entity-based type for measure nouns
- no change in the semantics of prepositions
- choice of prepositions derived from one-dimensionality and inherent verticality

5. **The bigger picture**

Measure phrases have been investigated or mentioned in the following environments:

(i) pseudo-partitives
(ii) arguments of measure verbs, such as *weigh* or *last* (Adger 1994)

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2 https://cathyjf.com/articles/effect-of-capitalisation
(iii) small clauses, as in *The temperature is ninety and rising* (Montague 1973, but not about measures at all); perhaps including appositive genitives, as in *the height of 5 km*

(iv) measure phrases with AP predicates, as in *five meters long(er)*

(v) measure phrases in PPs, as in *five meters above the house* (Zwarts 1997)

Unification attempts:


Strikingly, all these environments allow prepositional measures (13)

All standard approaches must link transitive vs. intransitive uses of measure nouns. For us it is a side effect of them being containers

6. **APPENDIX I: THE SEMANTICS OF MEASURE SPACES**

Everyone has to assume some semantics for measure nouns

What needs to be there:

- the dimension of measurement (length vs. weight)
- the unit of measurement (meter vs. kilometer)

More controversially:

- the number
- the substance being measured (Grosu and Landman 1988; see the appendix)
- the entity measured itself (Grosu and Landman 1988; see the appendix)

6.1. **Prior approaches to measure pseudo-partitives**

Various assumptions about the complexity of scale structure: what is incorporated into the notion of a scale

Ojeda 2003: measure phrases as equivalence classes (not compositional)

The parameter of variation (degree) can be just a **number**, with measurement implicit in the semantics of measure nouns:

\[
\begin{align*}
\text{(15) a. } & [\text{liter}] = \lambda n. \lambda P. \lambda x. (P(x) \land \text{liter}'(x) = n) & \text{Krifka 1990} \\
\text{b. } & [\text{kilo}] = \lambda k. \lambda n. \lambda x. (k(x) \land \mu \text{kg}(x) = n) & \text{Scontras 2014}
\end{align*}
\]

Existential quantification to derive intransitive measure phrases

Approaches built on the basis of adjectival scalarity (Schwarzschild and Wilkinson 2002, Schwarzschild 2005, etc.) incorporate the **dimension** of the measurement into the scale itself, with different scales for weight and size. To account for the pseudo-partitive use, a measure function should be added that is external to the syntax of measure nouns:

\[
\begin{align*}
\text{(16) a. } & \text{Three ounces of gold disappeared.} & \text{Schwarzschild 2005} \\
\text{b. } & \exists x [\text{gold}(x) \land \text{disappeared}(x) \land \text{three-ounces}(\text{wt.}(x))] \\
\text{(17) a. } & \text{three kilos of books} & \text{Rothstein 2011a} \\
\text{b. } & \lambda x. x \in *(\text{BOOK}_\text{root} \cap k) \land \text{MEAS}(x) = \langle \text{KILO}, 3 \rangle
\end{align*}
\]

Further complexity: including the entity being measured and the measurement unit:

NB: See section 9 for an argument against Grosu and Landman’s analysis

\[
\text{(18) } [\text{three}] = \lambda P. \lambda x. (P(x) \land \text{DEGREE}_P(x) = \langle 3, P, x \rangle) & \text{Grosu and Landman 1988}
\]
The richer conceptualization of scale structure places the compositional structure of pseudo-partitives into the ontology.

Our proposal: measure nouns denote containers, therefore the measure function is derived.

### 6.2. Spatial semantics

19) Regular spatial ontology (based on Zwarts and Winter 2000)
   a. $V$ is a vector space with appropriate operations and properties (vector addition, scalar multiplication)
   b. $D_p = V$ are points (variables $p$, $q$) (represented as single vectors w.r.t. origin $O$)
   c. $D_L = V \times V$ are located vectors (variables $u$, $v$, $w$) (represented as pairs of vectors, e.g. $(v, w)$)
   d. if $u = (v, w)$, then $E\text{-POINT}(u) = v + w$ (the end-point of a located vector)
   e. (convex) object has eigenspace (Wunderlich 1991) (set of points from $D_p$ that it occupies)
   f. $\text{LOC}_{s\text{pt}}$ gives spatial boundary of eigenspace of object (Zwarts and Winter 2000: whole eigenspace. NB: All their prepositions can naturally be redefined in the terms of the boundary. It remains an open question whether there are any prepositions or other natural language predicates that make reference only to the eigenspace.)
   g. $up \in V$ is a vector of arbitrary length representing the upward direction

20) Compositional process: over the rock
   a. $[\text{the rock }]$
      the-rock $= \text{tx . rock } (x)$
   b. $[\text{LOC } [\text{the rock }]$
      LOC(\text{the-rock}) (boundary of the rock)
   c. $[\text{over } [\text{LOC the rock }]$
      OVER(LOC(\text{the-rock}))
      $= \lambda v. \text{EXT}(v, \text{LOC(\text{the-rock})) \land UP(v)$
      (vectors pointing up from the boundary of the rock)
   d. $[\text{LOC}− [\text{over LOC the rock }]]$
      LOC−(OVER(LOC(\text{the-rock))))
      $= \lambda x. \forall p \in \text{LOC}(x) \exists v [\text{EXT}(v, \text{LOC(\text{the-rock})) \land UP(v) \land E\text{-POINT}(v)=p]$ (objects positioned at such vectors)

21) Definitions of spatial primitives
   a. $\text{LOC}− =_\text{def} \lambda W, \lambda x. \forall p \in \text{LOC}(x) \exists v \in W [E\text{-POINT}(v)=p]$ (maps set of vectors to set of objects of which the boundary points are located at those vectors)
   b. $\text{EXT}(v,A)$; $v$ points outward from the boundary of $A$ (no lengthening of $v$ reaches the boundary of $A$ again)
   c. $\text{UP}(v)$ is short for $c(up,v) > |v|_{uw}$ (v points more upwards than sideways)
d. \( \text{DOWN}(v) \) is short for \( c(-up,v) > |v| \)

e. \((v \text{ points more down than sideways})\)
\begin{align*}
\text{SHORT}(v) & \text{ means } |v| < r_1 \\
(\text{the length of } v \text{ is smaller than a small number } r_1)
\end{align*}

f. \( \text{INTER}(v,A,B) := \text{E-POINT}(v) \in \text{CO}(A \cup B) \setminus A \setminus B \) 
\begin{align*}
\text{CO}(A \cup B) \setminus A \setminus B & = \text{convex hull of regions } A \text{ and } B \text{ themselves} \\
(\text{v points from } A \text{ to } B \text{ or from } B \text{ to } A)
\end{align*}

(22) **Lexical definitions**

\begin{align*}
\text{OVER} & := \text{def} \lambda A. \lambda v. \text{EXT}(v,A) \land \text{UP}(v) \\
\text{UNDER} & := \text{def} \lambda A. \lambda v. \text{EXT}(v,A) \land \text{DOWN}(v) \\
\text{AROUND} & = \text{NEAR} := \text{def} \lambda A. \lambda v. \text{EXT}(v,A) \land \text{SHORT}(v) \\
\text{BETWEEN} & := \text{def} \lambda A. \lambda B. \lambda v. [\text{EXT}(v,A) \lor \text{EXT}(v,B)] \land \text{INTER}(v,A,B)
\end{align*}

To this we add what we know about containers

6.3. **Measure nouns as abstract containers**

First stab at regular containers:
- material outside and functional inside; inherent boundary
- filling establishes a natural mapping from content (substance) to vertical level
- filling to the proper level (‘full’) establishes a unit of volume
- the same for different filling substances

(23) **Concrete containers: filling and containing**

a. \( \text{FILL} \left(c, s\right) := \text{container } c \text{ contains substance } s \)
\begin{align*}
(\text{i}) & \text{ s corresponds to a kind} \\
(\text{ii}) & \text{implies:} \\
(\text{a}) & \text{there is an } x, \text{ R}(x,s) \text{ (Carlsonian realization)} \\
(\text{b}) & x \text{ ‘fills’ } c \text{ to the proper level, so that } c \text{ measures out a quantity } x \text{ of } s
\end{align*}

b. \( \llbracket \text{CONT} \rrbracket = \lambda P.e. \lambda k. \lambda c. P(c) \land \text{FILL}(c,k) \)

c. \( \llbracket \text{CONT} \rrbracket = \lambda C.e. \lambda x. \exists c. \exists k. C(c) \land \text{R}(x,k) \land \text{FILL}(c,k) \)

NB: As the containment function should be applicable to an entity, perhaps CONT \(^{-}\) is its plural version

The containment relation FILL is not a primitive and can be idealized in spatial terms (every point of the object internal to its boundary is the relevant substance)

NB: The fact that the definition is spatial is fascinating in itself, but may be of relevance in the future

(24) **Abstract containers**

a. **Concrete space is reduced to the most abstract (measure) space**
\begin{align*}
(\text{i}) & \text{three dimensions > one dimension (vertical)} \\
(\text{ii}) & \text{many types of substances > one type of substance} \\
(\text{iii}) & \text{one origin} \\
(\text{iv}) & \text{obligatory grounding (bottom of an object is either zero or the top of another)}
\end{align*}

b. \( C \subset D_e := \text{abstract containers} \)
\begin{align*}
(\text{i}) & \text{units of measurement for different dimensions (weight, volume, ...)} \\
(\text{ii}) & \text{located in abstract spaces} \\
(\text{iii}) & \text{pounds, ounces, grams are located in the same ‘weight’ spaces} \\
(\text{iv}) & \text{pounds of different substances in different ‘weight’ spaces}
\end{align*}

c. **for every container**
\begin{align*}
(\text{i}) & \text{eigenspace: one-dimensional set of points } \{s \cdot up: 0 \leq s \leq h \} \text{ for some } h \in \mathbb{R}
\end{align*}
(ii) ranging from $O(0\text{-up})$, the ‘base’ of the container
(iii) to some point $h$-up, the ‘top’ of the container
(iv) LOC$(c) = \{h$-up$\}$, because the base is non-functional
(idealization as point, cf. Herskovits 1986)

d. two abstract containers $c_1$ and $c_2$ in the same space
(i) are indistinguishable if they can be superimposed (bottom at 0, same height)
(ii) can be stacked on top of each other, which corresponds to addition of their
boundary vectors:
(a) stacking $c_1$ (with boundary $\{d_1\}$) and $c_2$ (with $\{d_2\}$) gives a sum of
containers with boundary $\{d_1+d_2\}$ (vector addition)
(b) for $n$ containers with the same boundary $\{d\}$, the boundary of their
stacking is $\{n\cdot d\}$ (scalar multiplication)

We will use a simplified representation for spatial composition that does not index either the
dimension or the substance
NB: Technically, vectors should be the Cartesian product of $V$ with the relevant dimension and with the relevant
substance, e.g., $V \times \text{weight} \times \text{jam}$. The dimension explains the incommensurability of, for instance, weight degrees
and volume degrees, while the substance accounts for, among other things, the previously unnoticed infelicity of
#between a pound of jam and a kilo of butter.

Instantiation:
> abstract containers can be instantiated
> concrete entities can be mapped to a variety of abstract containers
> existential quantification over abstract containers does not entail existence of their
realizations

Difference between concrete and abstract containers: the latter are located in a 1D space, yet
can be instantiated in the normal 3D space

(25) Compositional process: over a pound of jam

a. pound
   pound
   (a set of abstract containers located in a weight space of the same unit)

b. jam
   jam
   (the jam kind, sort within type $e$)

c. $[\text{CONT pound}]$
   $\lambda x, y. \text{pound}(y) \land \text{FILL}(y, x)$
   (function from kinds to abstract containers containing portions of that kind)

d. $[\text{CONT pound}]$ of jam
   $\lambda y. \text{pound}(y) \land \text{FILL}(y, \text{jam})$
   (set of abstract pound containers filled with portions of jam)

e. $[\text{a [[CONT pound] of jam]]]$
   $f(\lambda y. \text{pound}(y) \land \text{FILL}(y, \text{jam}))$
   ($f$ is a choice function picking out one arbitrary jam-filled pound container)

f. $[\text{LOC [a [[CONT pound] of jam]]}]$
   LOC$(f(\lambda y. \text{pound}(y) \land \text{FILL}(y, \text{jam})))$
   (boundary of eigenspace occupied by that one-pound container filled with jam)

g. $[\text{over [LOC [a [[CONT pound] of jam]]]]}$
   $\text{OVER}(\text{LOC}(f(\lambda y. \text{pound}(y) \land \text{FILL}(y, \text{jam})))) =$
   $\lambda v. \text{EXT}(v, \text{LOC}(f(\lambda y. \text{pound}(y) \land \text{FILL}(y, \text{jam})))) \land \text{UP}(v)$
Ora Matushansky & Joost Zwarts
Making space for measures

(set of vectors pointing upward from the top of the pound container filled with jam)

h. \( \text{LOC}^- \left( \text{over} \left[ \text{LOC} \left[ \text{a} \left[ \text{CONT} \text{pound} \right] \text{of jam} \right] \right] \right) \)
\( \text{LOC}^- \left( \lambda \mathbf{v}. \text{EXT}(\mathbf{v}, \text{LOC}(f(\lambda y_{-}. \text{pound}(y) \land \text{FILL}(y, \text{jam})))) \land \text{UP}(\mathbf{v}) \right) \)
(set of abstract containers of which the top is located at a vector pointing upward from the top of the pound container filled with jam)

i. \( \text{CONT}^- \left( \text{LOC}^- \left( \text{over} \left[ \text{LOC} \left[ \text{a} \left[ \text{CONT} \text{pound} \right] \text{of jam} \right] \right] \right) \right) \)
\( \text{FILL}^- \left( \text{LOC}^- \left( \lambda \mathbf{v}. \text{EXT}(\mathbf{v}, \text{LOC}(f(\lambda y_{-}. \text{pound}(y) \land \text{FILL}(y, \text{jam})))) \land \text{UP}(\mathbf{v}) \right) \right) \)
(the set of quantities of jam that fill up abstract weight containers that are higher than one pound)

(26) under five liters

a. liter : \text{liter}
(set of abstract liter containers)

b. five : \( \lambda P_{\infty}, \lambda x_{\infty}, \exists S_{\infty} \left[ \Pi(S(x)) \land |S|=5 \land \forall s \in S.P(s) \right] \)
Ionin and Matushansky 2006

c. five liters : \( \lambda x_{\infty}, \exists S_{\infty} \left[ \Pi(S(x)) \land |S|=5 \land \forall s \in S.\text{liter}(s) \right] \)
(the set of entities that can be partitioned into five liter containers)

d. INDEF \( [\text{five liters}] : f(\lambda x_{\infty}, \exists S_{\infty} \left[ \Pi(S(x)) \land |S|=5 \land \forall s \in S.\text{liter}(s) \right] ) \)
(one arbitrary element from the previous set)

e. \( \text{LOC}^- \left[ \text{INDEF} \left[ \text{five liters} \right] \right] : \text{LOC}((f(\lambda x_{\infty}, \exists S_{\infty} \left[ \Pi(S(x)) \land |S|=5 \land \forall s \in S.\text{liter}(s) \right] )) \land \text{OVER}(\mathbf{v}) \)
(top boundary of a stack of five stacked liter containers)

f. under \( \left[ \text{LOC}^- \left[ \text{INDEF} \left[ \text{five liters} \right] \right] \right] : \lambda \mathbf{v}. \text{EXT}(\mathbf{v}, \text{LOC}(f(\lambda x_{\infty}, \exists S_{\infty} \left[ \Pi(S(x)) \land |S|=5 \land \forall s \in S.\text{liter}(s) \right] )) \land \text{DOWN}(\mathbf{v}) \)
(set of vectors that point down from the top boundary of the stack)

g. \( \text{LOC}^- \left[ \text{under} \left[ \text{LOC}^- \left[ \text{INDEF} \left[ \text{five liters} \right] \right] \right] \right] : \lambda \mathbf{v}. \text{EXT}(\mathbf{v}, \text{LOC}(f(\lambda x_{\infty}, \exists S_{\infty} \left[ \Pi(S(x)) \land |S|=5 \land \forall s \in S.\text{liter}(s) \right] )) \land \text{OVER}(\mathbf{v}) \)
(set of volume containers of which the boundary is located at the endpoint of a vector from the previous set)

6.4. Concrete pseudo-partitives

Core intuition: anything can be a container. Containing itself by default (\text{CONT})

Projection: the mapping of a concrete object into the relevant abstract space:
Weighing therefore amounts to determining the abstract container corresponding to the object in the weight-space.

It is completely irrelevant what that space is made of.

When we map kinds into abstract containers, these latter become standardized (the ad hoc measure of Partee and Borschev 2012)

When the dimension of the space being mapped into is irrelevant, but the substance (plural) it is made of is kept constant, we get cardinality.

7. **APPENDIX II: HEADENESS AND CONSTITUENCY IN PSEUDO-PARTITIVES**

Two major possibilities examined:

(27a) Klooster 1972, Selkirk 1977, Lehrer 1986, Vos 1999, Grimshaw 2007, Landman 2015, etc.: the measure noun is the head of the pseudo-partitive; the substance NP is merged as its sister (complement)

(27d) Gawron 2002, Rothstein 2009a, b, 2011a, b, etc.: the substance noun is the head of the pseudo-partitive; the measure phrase is merged as its specifier.

Really, two independent issues:

- headedness: which noun projects?
- constituency: does the measure noun form a constituent with the cardinal or with the substance NP?

(27) a. **measure head, cascade**

```
    Num  NP1
       /   \
      three N   PP
       /     \
     liters P  NP2
       \     / of
          water
```

b. **measure head, adjunction**

```
    Num  NP1    PP
       /   \
      three N   P  NP2
       /     \    \
     liters of  water
```

c. **substance head, cascade**

```
    xNP1  CIP1
       /   \
      three Cl  NP1
       /     \    / of
     liters  water
```

d. **substance head, specification**

```
    Num  NP1    NP2
       /   \
      three N   P  NP2
       /     \    \
     liters of  water
```

The structural approach to prepositional measures only works for the structures in (27b,d) and still requires *compositional semantics of combining a locative preposition with a measure phrase*.

7.1. **Measure noun as the head (against the structures in (27c,d))**

External syntax compatible with both views (corpus examples from Keizer 2007:122):

(28) a. ... nearly two million tons of crude **have** already been pumped into the sea.

b. Ten years of Mrs Thatcher **has** wiped out…
Internal syntax is not
The structures in (6c,d) have no room for the preposition of or for genitive case:
\[(29)\] kružka češskogo piva
\[mug\] Czech.GEN beer.GEN
\[mug of Czech beer\]

**External case-assignment** targets the measure noun:
\[(30)\] On prines butylku vodki.
\[he\] brought bottle.ACC vodka.GEN
\[He brought a bottle of vodka.\]

The substance NP may also be marked with the externally assigned case only if the measure noun is:
\[(31)\] a. na vaptisun mriadhes pistus/pistus
\[to baptise.PL thousands.ACC believers.ACC/GEN
\[to baptize thousands of believers\]
\[b. piva një shishe verë\]
\[drank.1SG a bottle.ACC=NOM wine.ACC=NOM
\[I drank a bottle of wine.\]

**NP-internal agreement** is with the measure noun (Ruys [to appear], cf. van Gestel 1986):
\[(32)\] een liter water die/*dat we gedronken hebben
\[a liter.C water.N that.C/N we drunk have
\[a liter of water that we drink\]

Ruys [to appear]: unification with **collective nouns**, which must head the partitive NP:
\[(33)\] The herd of zebras is/are grazing.
\[(34)\] a. een doos koekjes
\[a box cookies\]
\[a box of cookies\]
\[b. en gruppe turister\]
\[a group tourists\]

7.2. **Substance NP as the sister of the measure noun (against the structure in (27b))**

The central function of measure nouns in pseudo-partitives is that they measure a substance. Two potential **sources for the measuring relation**: argument structure and a functional head

No evidence cross-linguistically for such an extra functional head in pseudo-partitives

If **argument structure**, complex compositional semantics (Rothstein 2011a, Kennedy 2015) for the cardinal-measure constituent:
\[(35)\] \[\text{[kilo]} = \lambda n. x.\text{MEAS}(x) = \langle \text{KILO}, n\rangle\]

Rothstein 2011a

Unwelcome consequences: the cardinal is an argument of the measure noun

Ruys [to appear]: **cardinals can be absent in pseudo-partitives**:
\[(36)\] a/that/Eddy's liter of vodka

On the hypothesis that the cardinal and the measure noun form a constituent NP, the lack of a determiner inside this NP is unexpected, given that a bare measure NP is ungrammatical:
(37) This bottle holds *(one) liter.

If cardinals are treated as numbers (Kennedy 2015), a 
**covert many must be assumed in all numeral NPs** (cf. Hackl 2000)

**C-selection:** in Dutch and in Scandinavian (Delsing 1993, Hankamer and Mikkelsen 2008), bare pseudo-partitives are only allowed with NP substances, for DPs a true partitive must be used:

(38) a. en gruppe turister  
    a group toursists  
    *a group of tourists*  
    Danish, Hankamer and Mikkelsen 2008

b. en gruppe af turisterne  
   a group of tourists.DEF  
   *a group of the tourists*

The syntactic choice between NP vs. PP cannot be accommodated in (27b): adjuncts cannot be c-selected

Evidence for the substance noun as the head:

(39) a. ena oreo/kokino/malako zeyghari paputsia  
    a nice/red/comfy pair shoes  
    *a nice/red/comfy pair of shoes*  
    Stavrou 2003

b. a delicious box of Belgian chocolates

c. a nice warm cup of tea

(40) #one melted cup of icecream  
    Landman 2015

Frequent claim: the adjective actually modifies the substance NP

- this is merely metonymy (the pair is comfy, this box is delicious)
- with a true measure noun modification is impossible (Rothstein 2011a)

Rothstein 2011a: different syntax for measure and container readings:

(41) a. The waiter brought three expensive glasses of cognac.  
    Rothstein 2011a

b. She added three expensive glasses(ful) of cognac to the sauce.

Landman 2015: same head-complement syntax for measure and container readings, different modes of composition

Our view: **concrete vs. abstract readings of the container noun glass**, with only the former compatible with modification

7.3. Are prepositional measures an argument for the cardinal-measure constituency?

Problems to be resolved:

- **PP-internally:** the semantics of the preposition-MP combination
- **PP-externally:** the entity denotation and nominal syntax

(i) has to be resolved with any constituency

(ii) gives the right results only with the structure in (27d), the price being that *of* is ignored:
(42)  b. measure head, adjunction
      PP₁
      P  NP₁  P  NP₂
      about three liters of water

d. substance head, specification
      PP
      P  NP₁  [of] NP₂
      about three liters

Too little gain: a stipulation is needed to not have a PP as a result anyway

8. APPENDIX III: INTERNAL STRUCTURE OF PREPOSITIONAL NUMERALS

Divergent approaches to internal structure:
➢ Plank 2004: prepositions combine with the entire NP
➢ Corver and Zwarts 2006: prepositions combine with the numeral to the exclusion of the NP

Prepositional measures suggest that Corver and Zwarts cannot be right: the cardinal is not obligatory

Plank 2004: evidence from case-assignment: in many languages in his sample prepositional numerals are only allowed in direct case positions, where the NP surfaces in the case assigned by the preposition:

(43)  a. V institute obučaetsja około desjati tysjač studentov.
      Russian at institute studies around ten. GEN thousand. GEN students. GEN
      There are approximately ten thousand students studying at the institute.

b. meždu pjetju i desjetju litrami vody
      between five. INS and ten. INS liters. INS water. GEN
      between five and ten liters of water

The entire numeral NP is marked with the case assigned by the preposition

In German this option is available only when no external case is assigned (44b), otherwise the external case wins (44a):

(44)  a. mit gegen Hundert Arbeiter-n
      German, Plank 2004
      with towards hundred worker-DAT.PL
      with approximately hundred workers

b. Fischel’s Verschwinden gegen ein-en Monat nach Ostern
      Fischel’s disappearance around one-MSG. ACC month after Easter
      Fischel’s disappearance at approximately one month after Easter.

There doesn’t seem to be a structural ambiguity distinguishing between locative and measure readings of the same prepositions

9. APPENDIX IV: DEGREE RELATIVES

Grosu and Landman 1988: a richer notion of a degree is necessary

Starting point: degree relatives (Carlson 1977, Heim 1987):

(45)  a. I took with me the three books that/Ø there were __ on the table.

b. #I took with me the three books which there were __ on the table.

Explanation for the contrast: the gap in (45b) is an e-type variable and definite, the gap in (45a) is a d-type variable (a degree) and forms part of an indefinite NP:
The identity of substance is not necessary:

(47) a. It will take us the rest of our lives to drink the champagne that they spilled that evening.

b. We will never be able to recruit the soldiers that the Chinese paraded last May Day.

The relative is interpreted as a comparative.

Grosu and Landman 1988: comparative suppletion is impossible, unlike in a comparative:

(48) a. *It will take us the rest of our lives to drink the champagne that they spilled beer that evening.

b. It will take us the rest of our lives to drink as much champagne as they spilled beer that evening.

The sortal has to be present in the relative clause.

Grosu and Landman 1988: head-internal analysis, which is later rendered superfluous by the semantics.

The identity of substance reading cannot be derived via the degree analysis as long as degrees are defined as sets of numbers on a scale (here, the cardinality scale).

(50) I took with me every book that there was on the table. \( \checkmark \) substance, \( \times \) quantity

M&Z: (50) cannot plausibly be analyzed as a degree reading: what does every quantify over?

### 9.1. Complex degrees

Grosu and Landman 1988: degrees must keep track of what they measure:

(51) \( [\text{three}] = \lambda P \lambda x . P(x) \land \text{DEGREE}_P(x) = \langle 3, P, x \rangle \)  

Grosu and Landman 1988

Other measures not discussed, but clear compositional problems for three meters.

The denotation of the relative clause is then:

(52) \( \lambda d . d = \max \{ \langle |x|, \text{books}, x \rangle : \text{books}(x) \land \text{on-the-table}(x) \} \)

How does this compose with books in the main clause?

Two additional operations postulated: SUBSTANCE (applying to the degree CP and extracting the substance measured) and X (composing the degree CP with the head NP as three does), to derive the two meanings.

Problems:

\( \triangleright \) totally ad hoc


➢ requires a null *many* in the degree CP and sometimes of its equivalent in the head NP
➢ does not extend to any other measure nouns
➢ does not work for *every*

Are degree relatives actually about degrees?

9.2. Maximization

Butler 2001: relative clauses can be interpreted restrictively or exhaustively

(53) Peter ate everything that would fit in his pocket.

**Restrictive reading:**
Peter ate everything (relevant) that was of an appropriately small size.
\[\forall x(P(x) \rightarrow A(x))\]

**Exhaustive reading:**
Peter ate a pocketfull of something.
\[\exists x(P(x) \land \neg\exists y [x \neq y \land P(y) \land \Box(P(y) \rightarrow P(x))] \land A(x))\]

Dynamic-semantic account assuming that relative clauses introduce discourse referents which can then be equated with the head:

(54) a. restrictive reading:
for every thing that Peter ate it would fit in his pocket

b. exhaustive reading:
what Peter ate would fit in his pocket

Non-compositional, the determiner simply ignored

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