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MAKING SPACE FOR MEASURES
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## 1. Introduction: PREPOSITIONAL MEASURE PHRASES

Starting point: the syntax and compositional semantics of examples like (1):
(1) a. Don't touch the steering wheel if you have drunk over five glasses of wine.
b. I ate around a pound of jam.
c. The mass of the meteorite was estimated at under 66 tons.
d. I was swimming between a kilometer and a mile four days a week.

Issues to be discussed and hopefully resolved:
> internal semantics: the normal semantics of spatial prepositions requires an entitydenoting internal argument. However, $e$ is not the usually assumed denotation for measure phrases, especially if they do not combine with a substance NP (1c,d)
> external syntax and semantics: the output of spatial PPs is generally not taken to be an entity and a PP is generally not a good substitute for an argument NP
The core of the proposal: measure nouns denote abstract containers located in a vertically oriented half-open one-dimensional space. Consequences:
$>$ measure phrases are entities and can therefore combine with prepositions
$>$ vertical orientation follows from the container concept
> algebra of scalar addition and multiplication, i.e., the scalar structure, follows from the properties of one-dimensional space
$>$ the interpretation of spatial prepositions is unchanged
$>\quad$ constraints on the choice of prepositions are explained
$>$ unified approach to abstract and concrete pseudo-partitives becomes possible
> lack of other dimensions makes it possible to account for the container-content ambiguity noted for pseudo-partitives (Selkirk 1977, Landman 2004, Grimshaw 2007, Rothstein 2009a, Partee and Borschev 2012, Duek and Brasoveanu 2015, etc.),
The bigger picture: reconstructing degrees as entities in concrete 1D space without the need to postulate a special semantic type or sort

## 2. THE SPECIFICS OF THE PROBLEM

Many different approaches to the semantics of spatial prepositions (Wunderlich 1991, Zwarts and Winter 2000, Kracht 2002, Bateman, Hois, Ross and Tenbrink 2010, etc.)

### 2.1. The syntax of the preposition

To the best of our knowledge, almost no prior work on measure uses of spatial prepositions
The semantics and pragmatics of up to (Nouwen 2008, 2010, Schwarz, Buccola and Hamilton 2012, Blok 2013, 2016, [to appear]): connection to the directional preposition exploited, but not derived
The syntax and semantics of prepositional numerals (Plank 2004, Corver and Zwarts 2006):
(2) Ik reken op [rond de twintig kinderen].

Dutch I count on round the twenty children I count on approximately twenty children.
Corver and Zwarts 2006: structural ambiguity:
(3) a. [around two] books

Problem for this approach: prepositional measures without a cardinal:
(4) a. around a pound
b. between a kilometer and a mile

Potential extension: structural ambiguity with prepositional pseudo-partitives -- only on the assumption that the cardinal and the measure noun form a constituent:
(5)

spatial reading/structure

Claim: assuming structural ambiguity does not solve anything
$>$ doesn't help with prepositional measures, like (4)
$>\quad$ or with the measure PPs without a measure noun (about twenty books)
$>$ still requires the explanation of how prepositions combine with measure NPs
> and with cardinals
$>$ predicts additional structural ambiguity for PP measures containing a cardinal
Plank 2004 for the syntax of the preposition

### 2.2. The syntax of pseudo-partitives

Klooster 1972, Selkirk 1977, Lehrer 1986, Vos 1999, Grimshaw 2007, Landman 2015, Ruys [to appear]:


NB: the label Num is used for expository purposes only; no positive commitment to the mode of combining a cardinal with its sister is required at this point. We will assume the cascade structure of Ionin and Matushansky 2006
NB: Semantically the simplest structure is with the substance PP merging higher than the approximative P -syntactically highly unlikely
Measure noun as the head: NP-internal agreement (Ruys [to appear], cf. van Gestel 1986):
(7) a. die éne liter water Dutch, Ruys [to appear] that.C one liter.C water.N that one liter of water
b. het onsje cocaïne
the.N metric.ounce.DIM.N cocaine.
the ounce of cocaine

Head-complement relation: visible construct state morphology for container nouns:
(8) šloša bakbukey yayin

Hebrew, Rothstein 2011a three bottles.cs wine three bottles of wine

However, no visible construct state morphology with measure nouns; assuming the same structure, further stipulations are needed one way or the other
Also: case-assignment to and inside the pseudo-partitive, c-selection, semantic composition (see appendix)

### 2.3. The issue of locative semantics

The problem of prepositional measures arises with any constituency and labeling: How does a spatial preposition combine with a measure NP to yield a non-spatial reading?
And of course, the substance NP does not have to be present, nor does the numeral:
(9) a. The temperature is below $20^{\circ} \mathrm{F}$.
b. We drank over a liter of vodka.

For the view that treats measure phrases as denoting in a separate domain (degrees) changing the meaning of the relevant prepositions is required

Core intuition: the locative metaphor (Lakoff and Johnson 1980, Lakoff and Núñez 2000): degrees can be metaphorically interpreted as positions on the vertical scale (see Plank 2004, Nouwen 2016 for numerals)
Formalization is still required for the mapping between the normal denotation of measure phrases and their metaphorical interpretation
Our proposal: there's no mapping - measures denote in the same domain as other sortals except for being 1D

## 3. Measure nouns as abstract containers

a. The picture is over the mantel. over expresses a vertical relation between two material objects in 3D space
b. I ate over a pound of jam. over expresses a vertical relation between two abstract containers in 1D space


### 3.1. The concept of an abstract container

Stereotypical properties of concrete containers:
$>$ verticality: a container must be vertical to contain substances
> measurement: a container can map different substances to the same volume unit
Natural properties of abstract containers:
> conceptualized as one-dimensional
$>$ no distinction between container and content (due to one-dimensionality)
> generalized to all quantities (weight, length, ...)
$>$ share one natural zero point (the "ground"; cf. Nouwen 2016), differ in height
abstract containers can be stacked on top of each other
> two abstract containers with the same height and content are indistinguishable unless stacked

### 3.2. Available spatial building blocks (simplified; see Appendix I)

Spatial building blocks in vector-space semantics (Zwarts and Winter 2000)
Two types in addition to $e$ and $t$ :
$>\quad$ type $p$ of points
$>\quad$ type $v$ of vectors, represent relations between points
Functions in spatial semantics:
LOC maps an entity to its spatial boundary* (type $\langle p, t\rangle$ )
$>$ a preposition maps a boundary to a particular set of vectors (type $\langle v, t\rangle)$
$>\mathrm{LOC}^{-}$maps a set of vectors to the set of entities (type $\langle e, t\rangle$ ) that are located at those vectors
(* the only adjustment necessary to the vector space semantics, notational variant for 3D, but crucial for 1D, part of the general schematization/idealization operations in spatial language, cf. Herskovits 1986)


### 3.3. Working with containers (simplified; see Appendix)

Two necessary functions in container semantics:
$>\quad$ CONT uses objects as containers filled with a substance (kind)
e.g., CONT(jar)(jam) = set of jars filled with jam
e.g., $\operatorname{CONT}($ pound $)($ jam $)=$ set of pounds filled with jam
$>\mathrm{CONT}^{-}$maps full containers to their contents
e.g., $\operatorname{CONT}^{-}(\operatorname{CONT}(\mathbf{j a r})(\mathbf{j a m}))=$ set of jam portions in jars
e.g., $\operatorname{CONT}^{-}(\operatorname{CONT}($ pound $)($ jam $))=$ set of jam portions of a pound

Abstract containers in 1D space:
> for every container $c, \operatorname{LOC}(c)$ is the singleton boundary containing only the top point (because the bottom point is always zero and therefore irrelevant)
$>$ for every region of vectors $R, \operatorname{LOC}^{-}(R)$ returns the set of containers of which the top coincides with a vector's endpoint
$>$ stacking of containers $\sim$ vector addition of their boundaries, multiple stacking $\sim$ scalar multiplication
> one dimension (vertical), one substance, one origin, obligatory grounding (the bottom of an object is either zero or the top of another)

### 3.4. The compositional structure

Of the phrase over a pound of jam with the meaning 'the set of portions of jam whose volume exceeds one pound':



CONT pound of pound
 jam


Loc $a \quad$ over $a$ pound of pound of jam jam jam jam

Remarks about CONT and CONT ${ }^{-}$:
$>$ the core meaning of measure nouns may be transitive, in which case CONT is not necessary
$>$ a structural representation of LOC, CONT and their reversals is not necessary, but CONT ${ }^{-}$may have a structural syntactic representation corresponding to a conversion from a PP to an NP (i.e., as a nominalizer):
(12) a. the [over 9 million liters of water and 50,000 filters distributed by FEMA] ${ }^{1}$
b. for the duration of those up to ten minutes ${ }^{2}$

[^0](13) a. Over 5 inches of snow could fall on Sunday.
b. The spaceship weighed well under a ton thanks to antigravity.
c. The rate is already below $\mathbf{7 \%}$.
d. The wait can be between two and three hours longer than anticipated.
e. The body was found around five meters behind the house.

The meaning of phrases with intransitive measure nouns and numerals can be derived with the assumptions of 3.2 and 3.3:
a. under five liters
b. [ LOC- [under [LOC [INDEF [five liters]]]]]
c. 'the set of volume containers of which the top coincides with the endpoint of a vector that points downward from the top of a stack of five one-liter containers'


## 4. SUMMARY

Core stipulation: the existence of abstract spaces with reduced structure
Independent evidence: spatial prepositions with result predicates (change from a prince into a frog) and result states (loving me to death/into an early grave)
Consequences:
the standard entity-based type for measure nouns
> an independently motivated implementation of scalar structure as spatial structure (cf. Faller 1998, 2000, Winter 2005, and for a wider perspective Gärdenfors 2004, 2014), without degrees or scales
$>$ no change in the semantics of prepositions
> choice of prepositions derived from one-dimensionality and inherent verticality

## 5. The bigGer picture

Measure phrases have been investigated or mentioned in the following environments:
(i) pseudo-partitives
(ii) arguments of measure verbs, such as weigh or last (Adger 1994)

[^1](iii) small clauses, as in The temperature is ninety and rising (Montague 1973, but not about measures at all); perhaps including appositive genitives, as in the height of 5 km
(iv) measure phrases with AP predicates, as in five meters long(er)
(v) measure phrases in PPs, as in five meters above the house (Zwarts 1997)

Unification attempts:
> (i) and (iv): Schwarzschild and Wilkinson 2002, Schwarzschild 2005, Rothstein 2011a, Scontras 2014
> (v) from (iv): Faller 1998, 2000, Winter 2005
Strikingly, all these environments allow prepositional measures (13)
All standard approaches must link transitive vs. intransitive uses of measure nouns. For us it is a side effect of them being containers

## 6. APPENDIX I: THE SEMANTICS OF MEASURE SPACES

Everyone has to assume some semantics for measure nouns
What needs to be there:
$>$ the dimension of measurement (length vs. weight)
$>\quad$ the unit of measurement (meter vs. kilometer)
More controversially:
> the number
> the substance being measured (Grosu and Landman 1988; see the appendix)
> the entity measured itself (Grosu and Landman 1988; see the appendix)

### 6.1. Prior approaches to measure pseudo-partitives

Various assumptions about the complexity of scale structure: what is incorporated into the notion of a scale

Ojeda 2003: measure phrases as equivalence classes (not compositional)
The parameter of variation (degree) can be just a number, with measurement implicit in the semantics of measure nouns:
a. $\quad \llbracket$ liter $\rrbracket=\lambda n \lambda \mathrm{P} \lambda \mathrm{x}\left[\mathrm{P}(\mathrm{x}) \wedge \operatorname{liter}^{\prime}(\mathrm{x})=\mathrm{n}\right]$

Krifka 1990
b. $\quad \llbracket \mathrm{kilo} \rrbracket=\lambda \mathrm{k} \lambda \mathrm{n} \lambda \mathrm{x} . \cup_{\mathrm{k}}(\mathrm{x}) \wedge \mu_{\mathrm{kg}}(\mathrm{x})=\mathrm{n}$

Scontras 2014
Existential quantification to derive intransitive measure phrases
Approaches built on the basis of adjectival scalarity (Schwarzschild and Wilkinson 2002, Schwarzschild 2005, etc.) incorporate the dimension of the measurement into the scale itself, with different scales for weight and size. To account for the pseudo-partitive use, a measure function should be added that is external to the syntax of measure nouns:
(16) a. Three ounces of gold disappeared.

Schwarzschild 2005
b. $\quad \exists \mathrm{x}[\operatorname{gold}(\mathrm{x}) \&$ disappeared( x$) \&$ three-ounces(wt.(x))]
(17) a. three kilos of books

Rothstein 2011a
b. $\quad \lambda \mathrm{x} . \mathrm{x} \in *\left(\mathrm{BOOK}_{\text {root }} \cap \mathrm{k}\right) \wedge \operatorname{MEAS}(\mathrm{x})=\langle\mathrm{KILO}, 3\rangle$

Further complexity: including the entity being measured and the measurement unit:
NB: See section 9 for an argument against Grosu and Landman's analysis

$$
\begin{equation*}
\llbracket \text { three } \rrbracket=\lambda \mathrm{P} \lambda \mathrm{x} . \mathrm{P}(\mathrm{x}) \wedge \mathbf{D E G R E E}_{\mathbf{P}}(\mathrm{x})=\langle 3, \mathrm{P}, \mathrm{x}\rangle \tag{18}
\end{equation*}
$$

The richer conceptualization of scale structure places the compositional structure of pseudopartitives into the ontology

Our proposal: measure nouns denote containers, therefore the measure function is derived

### 6.2. Spatial semantics

(19) Regular spatial ontology (based on Zwarts and Winter 2000)
a. $\quad V$ is a vector space with appropriate operations and properties (vector addition, scalar multiplication)
b. $\quad D_{p}=V$ are points (variables $\mathbf{p}, \mathbf{q}$ )
(represented as single vectors w.r.t. origin $O$ )
c. $\quad D_{v}=V \times V$ are located vectors (variables $\left.\mathbf{u}, \mathbf{v}, \mathbf{w}\right)$ (represented as pairs of vectors, e.g. $\langle v, w\rangle$ )
d. if $\mathbf{u}=\langle v, w\rangle$, then $\operatorname{E-Point}(\mathbf{u})=v+w$
(the end-point of a located vector)
e. (convex) object has eigenspace (Wunderlich 1991)
 (set of points from $D_{p}$ that it occupies)
f. $\quad \mathrm{LOC}_{e(p t)}$ gives spatial boundary of eigenspace of object (Zwarts and Winter 2000: whole eigenspace. NB: All their prepositions can naturally be redefined in the terms of the boundary. It remains an open question whether there are any prepositions or other natural language predicates that make reference only to the eigenspace.)
g. $\quad u p \in V$ is a vector of arbitary length representing the upward direction
(20) Compositional process: over the rock
a. [the rock]
the-rock (=ix.rock (x))
b. [ LOC [ the rock ]] LOC(the-rock)
(boundary of the rock)
c. [ over [ LoC the rock ]]

OVER(LOC(the-rock))
$x$
$\operatorname{LOC}(x) \operatorname{over}(\operatorname{Loc}(x)) \operatorname{LOC}-(\operatorname{over}(\operatorname{Loc}(x)))$
$=\lambda \mathbf{v} \cdot \operatorname{EXT}(\mathbf{v}, \operatorname{LOC}($ the-rock $)) \wedge \mathrm{UP}(\mathbf{v})$
(vectors pointing up from the boundary of the rock)
d. [ LOC- [ over LOC the rock ]]

LOC-(OVER(LOC(the-rock)))
$=\lambda x_{e} . \forall \mathbf{p} \in \operatorname{LOC}(x) \exists \mathbf{v}[\operatorname{EXT}(\mathbf{v}, \operatorname{LOC}($ the-rock $)) \wedge \mathrm{UP}(\mathbf{v}) \wedge \operatorname{E-POINT}(\mathbf{v})=\mathbf{p}]$
(objects positioned at such vectors)
(21) Definitions of spatial primitives
a. $\quad \operatorname{LOC}^{-}{ }^{\text {def }} \lambda W_{v t} \cdot \lambda x_{e} . \forall \mathbf{p} \in \operatorname{LOC}(x) \exists \mathbf{v} \in W[\mathrm{E}-\operatorname{POINT}(\mathbf{v})=\mathbf{p}]$
(maps set of vectors to set of objects of which the boundary points are located at those vectors)
b. $\quad \operatorname{EXT}(\mathbf{v}, A): \mathbf{v}$ points outward from the boundary of $A$
(no lengthening of $\mathbf{v}$ reaches the boundary of $A$ again)
c. $\quad \mathrm{UP}(\mathbf{v})$ is short for $c(u p, \mathbf{v})>\left|\mathbf{v}_{\perp u p}\right|$
( $\mathbf{v}$ points more upwards than sideways)
d. $\operatorname{DOWN}(\mathbf{v})$ is short for $c(-u p, \mathbf{v})>\left|\mathbf{v}_{\perp-u p}\right|$
e. ( $\mathbf{v}$ points more down than sideways)
$\operatorname{SHORT}(\mathbf{v})$ means $|\mathbf{v}|<r_{1}$
(the length of $\mathbf{v}$ is smaller than a small number $r_{1}$ )
f. $\operatorname{INTER}(\mathbf{v}, A, B): \operatorname{E-POINT}(\mathbf{v}) \in \mathrm{CO}(A \cup B) \backslash A \backslash B$
$\operatorname{CO}(A \cup B) \backslash A \backslash B=$ convex hull of regions $A$ and $B$ minus $A$ and $B$ themselves $(\mathbf{v}$ points from $A$ to $B$ or from $B$ to $A$ )
(22) Lexical definitions

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OVER \(=_{\text {def }} \quad \lambda A \cdot \lambda \mathbf{v} \cdot \operatorname{EXT}(\mathbf{v}, A) \wedge \mathrm{UP}(\mathbf{v})\)
\(\operatorname{UNDER}={ }_{\text {def }} \quad \lambda A \cdot \lambda \mathbf{v} \cdot \operatorname{EXT}(\mathbf{v}, A) \wedge \operatorname{DOWN}(\mathbf{v})\)
\(\operatorname{AROUND}=\mathrm{NEAR}={ }_{\text {def }} \lambda A \cdot \lambda \mathbf{v} \cdot \operatorname{EXT}(\mathbf{v}, A) \wedge \operatorname{SHORT}(\mathbf{v})\)
BETWEEN \(=_{\text {def }} \quad \lambda A . \lambda B . \lambda \mathbf{v}[\operatorname{EXT}(\mathbf{v}, A) \vee \operatorname{EXT}(\mathbf{v}, B)] \wedge \operatorname{INTER}(\mathbf{v}, A, B)\)
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To this we add what we know about containers

### 6.3. Measure nouns as abstract containers

First stab at regular containers:
$>$ material outside and functional inside; inherent boundary
$>$ filling establishes a natural mapping from content (substance) to vertical level
$>\quad$ filling to the proper level ('full') establishes a unit of volume
$>$ the same for different filling substances
(23) Concrete containers: filling and containing
a. $\quad$ FILL $(c, s):$ container $c$ contains substance $s$
(i) $s$ corresponds to a kind
(ii) implies:
(a) there is an $x, \mathrm{R}(x, s)$ (Carlsonian realization)
(b) $\quad x$ 'fills' $c$ to the proper level, so that $c$ measures out a quantity $x$ of $s$
b. $\quad \llbracket \mathrm{CONT} \rrbracket=\lambda P_{e t} \cdot \lambda k \cdot \lambda c . P(c) \wedge \operatorname{FILL}(c, k)$
c. $\quad\left[\mathrm{CONT}^{-} \rrbracket=\lambda C_{e t} \cdot \lambda x . \exists c \cdot \exists k . C(c) \wedge \mathrm{R}(x, k) \wedge \operatorname{FILL}(c, k)\right.$

NB: As the containment function should be applicable to an entity, perhaps $\mathrm{CONT}^{-}$is its plural version
The containment relation FILL is not a primitive and can be idealized in spatial terms (every point of the object internal to its boundary is the relevant substance)
NB: The fact that the definition is spatial is fascinating in itself, but may be of relevance in the future
(24) Abstract containers
a. Concrete space is reduced to the most abstract (measure) space
(i) three dimensions $>$ one dimension (vertical)
(ii) many types of substances > one type of substance
(iii) one origin
(iv) obligatory grounding (bottom of an object is either zero or the top of another)
b. $\quad C \subset D_{e}:$ abstract containers
(i) units of measurement for different dimensions (weight, volume, ...)
(ii) located in abstract spaces
(iii) pounds, ounces, grams are located in the same 'weight' spaces
(iv) pounds of different substances in different 'weight' spaces
c. for every container $c$
(i) eigenspace: one-dimensional set of points $\{\mathrm{s} \cdot u p: 0 \leq \mathrm{s} \leq h\}$ for some $\mathrm{h} \in \mathbb{R}$
(ii) ranging from $O(0 \cdot u p)$, the 'base' of the container
(iii) to some point $\mathrm{h} \cdot u p$, the 'top' of the container
(iv) $\operatorname{LOC}(c)=\{\mathrm{h} \cdot u p\}$, because the base is non-functional (idealization as point, cf. Herskovits 1986)
d. two abstract containers $c_{1}$ and $c_{2}$ in the same space
(i) are indistinguishable if they can be superimposed (bottom at 0 , same height)
(ii) can be stacked on top of each other, which corresponds to addition of their boundary vectors:
(a) stacking $c_{1}$ (with boundary $\left\{d_{1}\right\}$ ) and $c_{2}$ (with $\left\{d_{2}\right\}$ ) gives a sum of containers with boundary $\left\{d_{1}+d_{2}\right\}$ (vector addition)
(b) for $n$ containers with the same boundary $\{d\}$, the boundary of their stacking is $\{\mathrm{n} \cdot d\}$ (scalar multiplication)
We will use a simplified representation for spatial composition that does not index either the dimension or the substance
NB: Technically, vectors should be the Cartesian product of $V$ with the relevant dimension and with the relevant substance, e.g., $V \times$ weight $\times j a m$. The dimension explains the incommensurability of, for instance, weight degrees and volume degrees, while the substance accounts for, among other things, the previously unnoticed infelicity of \#between a pound of jam and a kilo of butter.

## Instantiation:

> abstract containers can be instantiated
concrete entities can be mapped to a variety of abstract containers
$>$ existential quantification over abstract containers does not entail existence of their realizations

Difference between concrete and abstract containers: the latter are located in a 1D space, yet can be instantiated in the normal 3D space
(25) Compositional process: over a pound of jam
a. pound
pound
(a set of abstract containers located in a weight space of the same unit)
b. jam
jam
(the jam kind, sort within type $e$ )
c. [ CONT pound ]
$\lambda x_{k} \cdot \lambda y_{c}$.pound $(y) \wedge \operatorname{FILL}(y, x)$
(function from kinds to abstract containers containing portions of that kind)
d. [ [ CONT pound ] of jam ]
$\lambda y_{c}$.pound $(y) \wedge \operatorname{FILL}(y, \mathbf{j a m})$
(set of abstract pound containers filled with portions of jam)
e. [ a [[ CONT pound ] of jam ]]
$f\left(\lambda y_{c}\right.$.pound $\left.(y) \wedge \operatorname{FILL}(y, \mathbf{j a m})\right)$
( $f$ is a choice function picking out one arbitrary jam-filled pound container)
f. [ LOC [ a [[ CONT pound ] of jam ]]]
$\operatorname{LOC}\left(f\left(\lambda y_{c}\right.\right.$. $\left.\left.\mathbf{p o u n d}(y) \wedge \operatorname{FILL}(y, \mathbf{j a m})\right)\right)$
(boundary of eigenspace occupied by that one-pound container filled with jam)
g. [over [LOC [ a [[ CONT pound ] of jam ]]]]
$\operatorname{OVER}\left(\operatorname{LOC}\left(f\left(\lambda y_{c} . \operatorname{pound}(y) \wedge \operatorname{FILL}(y, \mathbf{j a m})\right)\right)\right)=$
$\lambda \mathbf{v} \cdot \operatorname{EXT}\left(\mathbf{v}, \operatorname{LOC}\left(f\left(\lambda y_{c} \cdot \mathbf{p o u n d}(y) \wedge \operatorname{FILL}(y, \mathbf{j a m})\right)\right)\right) \wedge \operatorname{UP}(\mathbf{v})$
(set of vectors pointing upward from the top of the pound container filled with jam)
h. [ $\operatorname{LOC}^{-}$[ over [LOC [ a [[ CONT pound ] of jam ]]]]] $\operatorname{LOC}^{-}\left(\lambda \mathbf{v} \cdot \operatorname{EXT}\left(\mathbf{v}, \operatorname{LOC}\left(f\left(\lambda y_{c} . \operatorname{pound}(y) \wedge \operatorname{FILL}(y, \mathbf{j a m})\right)\right)\right) \wedge \operatorname{UP}(\mathbf{v})\right)$
(set of abstract containers of which the top is located at a vector pointing upward from the top of the pound container filled with jam)
i. [ $\mathrm{CONT}^{-}$[ $\mathrm{LOC}^{-}$[ over [ LOC [ a [[ CONT pound ] of jam ]] $]$] $]$] $\operatorname{FILL}^{-}\left(\operatorname{LOC}^{-}\left(\lambda \mathbf{v} \cdot \operatorname{EXT}\left(\mathbf{v}, \operatorname{LOC}\left(f\left(\lambda y_{c}\right.\right.\right.\right.\right.$. pound $\left.\left.\left.\left.\left.(y) \wedge \operatorname{FILL}(y, \mathbf{j} a m)\right)\right)\right) \wedge \operatorname{UP}(\mathbf{v})\right)\right)$ (the set of quantities of jam that fill up abstract weight containers that are higher than one pound)

pound

CoNT
pound of
jam

LOC $a$ pound of jam

over a pound of jam

LOC- over CONT- over a pound of a pound of jam jam

## under five liters

a. liter : liter
(set of abstract liter containers)
b. five : $\lambda P_{e t} \cdot \lambda x_{e} \cdot \exists S_{e t}[\Pi(S)(x) \wedge|S|=5 \wedge \forall s \in S . P(s)] \quad$ Ionin and Matushansky 2006
c. five liters : $\lambda x_{e} \cdot \exists S_{e t}[\Pi(S)(x) \wedge|S|=5 \wedge \forall s \in \operatorname{S.liter}(s)]$ (the set of entities that can be partitioned into five liter containers)
d. INDEF [five liters] : $f\left(\lambda x_{e} . \exists S_{e t}[\Pi(S)(x) \wedge|S|=5 \wedge \forall s \in S\right.$.liter $(s)$ ]) (one arbitrary element from the previous set)
e. LOC [INDEF [five liters]] : $\operatorname{LOC}\left(f\left(\lambda x_{e} \cdot \exists S_{e t}[\Pi(S)(x) \wedge|S|=5 \wedge \forall s \in S . \operatorname{liter}(s)]\right)\right)$ (top boundary of a stack of five stacked liter containers)
f. under [LOC [INDEF [five liters]]]:
$\lambda \mathbf{v} \cdot \operatorname{EXT}\left(\mathbf{v}, \operatorname{LOC}\left(f\left(\lambda x_{e} \cdot \exists S_{e t}[\Pi(S)(x) \wedge|S|=5 \wedge \forall s \in S . \operatorname{liter}(s)]\right)\right)\right) \wedge \operatorname{DOWN}(\mathbf{v})$ (set of vectors that point down from the top boundary of the stack)
g. LOC- [under [LOC [INDEF [five liters]]]]:
$\operatorname{LOC}-\left(\lambda \mathbf{v} \cdot \operatorname{EXT}\left(\mathbf{v}, \operatorname{LOC}\left(f\left(\lambda x_{e} \cdot \exists S_{e t}[\Pi(S)(x) \wedge|S|=5 \wedge \forall s \in S \cdot \operatorname{liter}(s)]\right)\right) \wedge \operatorname{DOWN}(\mathbf{v})\right)\right.$ (set of volume containers of which the boundary is located at the endpoint of a vector from the previous set)

### 6.4. Concrete pseudo-partitives

Core intuition: anything can be a container. Containing itself by default (CONT)
Projection: the mapping of a concrete object into the relevant abstract space:
> measuring: mapping for the relevant measure function
$>$ counting: mapping for the relevant space substance
Weighing therefore amounts to determining the abstract container corresponding to the object in the weight-space
It is completely irrelevant what that space is made of
When we map kinds into abstract containers, these latter become standardized (the ad hoc measure of Partee and Borschev 2012)
When the dimension of the space being mapped into is irrelevant, but the substance (plural) it is made of is kept constant, we get cardinality

## 7. APPENDIX II: HEADEDNESS AND CONSTITUENCY IN PSEUDO-PARTITIVES

Two major possibilities examined:
(27a) Klooster 1972, Selkirk 1977, Lehrer 1986, Vos 1999, Grimshaw 2007, Landman 2015, etc.: the measure noun is the head of the pseudo-partitive; the substance NP is merged as its sister (complement)
(27d) Gawron 2002, Rothstein 2009a, b, 2011a, b, etc.: the substance noun is the head of the pseudo-partitive; the measure phrase is merged as its specifier
Really, two independent issues:
headedness: which noun projects?
> constituency: does the measure noun form a constituent with the cardinal or with the substance NP?
(27) a
measure head, cascade

c. substance head, cascade

b. measure head, adjunction

d. substance head, specification


The structural approach to prepositional measures only works for the structures in (27b,d) and still requires compositional semantics of combining a locative preposition with a measure phrase

### 7.1. Measure noun as the head (against the structures in (27c,d)

External syntax compatible with both views (corpus examples from Keizer 2007:122):
(28) a. ... nearly two million tons of crude have already been pumped into the sea.
b. Ten years of Mrs Thatcher has wiped out...

Internal syntax is not
The structures in ( $6 \mathrm{c}, \mathrm{d}$ ) have no room for the preposition of or for genitive case:
(29) kružka češskogo piva

Partee and Borschev 2012 mug Czech.gEn beer.GEN mug of Czech beer
External case-assignment targets the measure noun:
(30) On prines butylku vodki.

Partee and Borschev 2012
he brought bottle.ACC vodka.GEN
He brought a bottle of vodka.
The substance NP may also be marked with the externally assigned case only if the measure noun is:

| a. na vaptisun mriadhes pistus/piston | Greek |
| :--- | :--- | :--- |
| to baptise.PL thousands.ACC believers.ACC/GEN |  |
| to baptize thousands of believers |  |

b. piva një shishe verë Albanian (Giusti and Turano 2004) drank.1SG a bottle.ACC=NOM wine.ACC=NOM I drank a bottle of wine.
NP-internal agreement is with the measure noun (Ruys [to appear], cf. van Gestel 1986):
(32) een liter water die/*dat we gedronken hebben Ruys [to appear]
a liter.C water. N that.c/ N we drunk have
a liter of water that we drink
Ruys [to appear]: unification with collective nouns, which must head the partitive NP:
(33) The herd of zebras is/are grazing.

Dodge and Wright 2002
(34) a. een doos koekjes

Dutch, Ruys [to appear] a box cookies a box of cookies
b. en gruppe turister

Danish, Hankamer and Mikkelsen 2008 a group tourists a group of tourists

### 7.2. Substance NP as the sister of the measure noun (against the structure in (27b))

The central function of measure nouns in pseudo-partitives is that they measure a substance. Two potential sources for the measuring relation: argument structure and a functional head
No evidence cross-linguistically for such an extra functional head in pseudo-partitives
If argument structure, complex compositional semantics (Rothstein 2011a, Kennedy 2015) for the cardinal-measure constituent:
(35) $\llbracket k i l o \rrbracket=\lambda n \lambda x \cdot \operatorname{MEAS}(x)=<$ KILO, $n>$

Rothstein 2011a
Unwelcome consequences: the cardinal is an argument of the measure noun
Ruys [to appear]: cardinals can be absent in pseudo-partitives:
(36) a/that/Eddy's liter of vodka

On the hypothesis that the cardinal and the measure noun form a constituent NP, the lack of a determiner inside this NP is unexpected, given that a bare measure NP is ungrammatical:
(37) This bottle holds *(one) liter.

If cardinals are treated as numbers (Kennedy 2015), a covert many must be assumed in all numeral NPs (cf. Hackl 2000)
C-selection: in Dutch and in Scandinavian (Delsing 1993, Hankamer and Mikkelsen 2008), bare pseudo-partitives are only allowed with NP substances, for DPs a true partitive must be used:
$\begin{array}{lll}\text { a. en gruppe turister } & \text { Danish, Hankamer and Mikkelsen } 2008 \\ \text { a group tourists } & \\ \text { a group of tourists } & \end{array}$
b. en gruppe af turisterne
a group of tourists.DEF a group of the tourists
The syntactic choice between NP vs. PP cannot be accommodated in (27b): adjuncts cannot be c-selected
Evidence for the substance noun as the head:
a. ena oreo/kokino/malako zevghari paputsia

Stavrou 2003 a nice/red/comfy pair shoes a nice/red/comfy pair of shoes
b. a delicious box of Belgian chocolates
c. a nice warm cup of tea
(40) \#one melted cup of icecream Landman 2015

Frequent claim: the adjective actually modifies the substance NP
$>$ this is merely metonymy (the pair is comfy, this box is delicious)
$>\quad$ with a true measure noun modification is impossible (Rothstein 2011a)
Rothstein 2011a: different syntax for measure and container readings:
(41) a. The waiter brought three expensive glasses of cognac.

Rothstein 2011a
b. \#She added three expensive glasses(ful) of cognac to the sauce.

Landman 2015: same head-complement syntax for measure and container readings, different modes of composition
Our view: concrete vs. abstract readings of the container noun glass, with only the former compatible with modification

### 7.3. Are prepositional measures an argument for the cardinal-measure constituency?

Problems to be resolved:
(vi) PP-internally: the semantics of the preposition-MP combination
(vii) PP-externally: the entity denotation and nominal syntax
(i) has to be resolved with any constituency
(ii) gives the right results only with the structure in (27d), the price being that of is ignored:
(42) b. measure head, adjunction

d. substance head, specification


Too little gain: a stipulation is needed to not have a PP as a result anyway

## 8. APPENDIX III: INTERNAL STRUCTURE OF PREPOSITIONAL NUMERALS

Divergent approaches to internal structure:
> Plank 2004: prepositions combine with the entire NP
> Corver and Zwarts 2006: prepositions combine with the numeral to the exclusion of the NP

Prepositional measures suggest that Corver and Zwarts cannot be right: the cardinal is not obligatory

Plank 2004: evidence from case-assignment: in many languages in his sample prepositional numerals are only allowed in direct case positions, where the NP surfaces in the case assigned by the preposition:
(43) a. V institute obučaetsja okolo desjati tysjač studentov. Russian at institute studies around ten.GEN thousand.GEN students.GEN There are approximately ten thousand students studying at the institute.
b. meždu pjatju i desjatju litrami vody between five.INS and ten.INS liters.INS water.GEN between five and ten liters of water

The entire numeral NP is marked with the case assigned by the preposition
In German this option is available only when no external case is assigned (44b), otherwise the external case wins (44a):
a. mit gegen Hundert Arbeiter-n

German, Plank 2004
with towards hundred worker-DAT.PL
with approximately hundred workers
b. Fischels Verschwinden gegen ein-en Monat nach Ostern

Fischel's disappearance around one-MSG.ACC month after Easter Fischel's disappearance at approximately one month after Easter.
There doesn't seem to be a structural ambiguity distinguishing between locative and measure readings of the same prepositions

## 9. APPENDIX IV: DEGREE RELATIVES

Grosu and Landman 1988: a richer notion of a degree is necessary
Starting point: degree relatives (Carlson 1977, Heim 1987):
(45) a. I took with me the three books that/ $\varnothing$ there were $\qquad$ on the table.
b. \#I took with me the three books which there were $\qquad$ on the table.

Explanation for the contrast: the gap in (45b) is an $e$-type variable and definite, the gap in (45a) is a $d$-type variable (a degree) and forms part of an indefinite NP:
(46) a. $\lambda d$. there are [d-many books] on the table
b. $\quad \lambda \mathrm{d} . \exists \mathrm{x}$ [books (x) \& $\mathrm{x} \mid=\mathrm{d} \&$ on-the-table ( x )]

The identity of substance is not necessary:
(47) a. It will take us the rest of our lives to drink the champagne that they spilled that evening.
b. We will never be able to recruit the soldiers that the Chinese paraded last May Day.
The relative is interpreted as a comparative
Grosu and Landman 1988: comparative suppletion is impossible, unlike in a comparative:
(48) a. *It will take us the rest of our lives to drink the champagne that they spilled beer that evening.
b. It will take us the rest of our lives to drink as much champagne as they spilled beer that evening.
The sortal has to be present in the relative clause
Grosu and Landman 1988: head-internal analysis, which is later rendered superfluous by the semantics
(49)


The identity of substance reading cannot be derived via the degree analysis as long as degrees are defined as sets of numbers on a scale (here, the cardinality scale)
(50) I took with me every book that there was on the table.

M\&Z: (50) cannot plausibly be analyzed as a degree reading: what does every quantify over?

### 9.1. Complex degrees

Grosu and Landman 1988: degrees must keep track of what they measure:

Other measures not discussed, but clear compositional problems for three meters
The denotation of the relative clause is then:
(52) $\lambda \mathrm{d} . \mathrm{d}=\max \{\langle | \mathrm{x} \mid$, books, x$\rangle$ : books $(\mathrm{x}) \wedge$ on-the-table $(\mathrm{x})\}$

How does this compose with books in the main clause?
Two additional operations postulated: SUBSTANCE (applying to the degree CP and extracting the substance measured) and X (composing the degree CP with the head NP as three does), to derive the two meanings
Problems:
> totally ad hoc
$>$ requires a null many in the degree CP and sometimes of its equivalent in the head NP
> does not extend to any other measure nouns
$>$ does not work for every
Are degree relatives actually about degrees?

### 9.2. Maximization

Butler 2001: relative clauses can be interpreted restrictively or exhaustively
(53) Peter ate everything that would fit in his pocket.

## Restrictive reading:

Peter ate everything (relevant) that was of an appropriately small size.
$\forall \mathrm{x}(\mathrm{P}(\mathrm{x}) \rightarrow \mathrm{A}(\mathrm{x}))$

## Exhaustive reading:

Peter ate a pocketfull of something.
$\exists \mathrm{x}(\mathrm{P}(\mathrm{x}) \wedge \neg \exists \mathrm{y}[\mathrm{x} \neq \mathrm{y} \wedge \mathrm{P}(\mathrm{y}) \wedge \square(\mathrm{P}(\mathrm{y}) \rightarrow \mathrm{P}(\mathrm{x}))] \wedge \mathrm{A}(\mathrm{x}))$
Dynamic-semantic account assuming that relative clauses introduce discourse referents which can then be equated with the head:
(54) a. restrictive reading:
for every thing that Peter ate it would fit in his pocket
b. exhaustive reading:
what Peter ate would fit in his pocket
Non-compositional, the determiner simply ignored

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[^0]:    ${ }^{1}$ https://www.whitehouse.gov/the-press-office/2016/05/03/fact-sheet-federal-support-flint-water-crisis-response-and-recovery

[^1]:    ${ }^{2}$ https://cathyjf.com/articles/effect-of-capitalisation

