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...WE CAN DO BETTER
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Much converging research: various kinds of expressions **in the scope of a modal** may have a **maximal or minimal reading**.

Rullmann 1995a: inferiority comparatives with a modal in the degree clause are ambiguous:

(1)   The helicopter was flying less high than a plane can fly.
      a.   the altitude of the helicopter is lower than the maximal altitude a plane can attain
      b.   the altitude of the helicopter is lower than the minimal altitude a plane can attain

Rullmann 1995b, Carston 1998, Kennedy 2010: **numerals under modals can have *at least*, *at most* or *exactly* readings**:

(2)   a.   In Britain, you have to be 18 to drive a car.   minimal
      b.   Once you have your degree, you can have a salary of $100,000 a year.

(3)   a.   She can have 2000 calories without putting on weight.   maximal
      b.   You may attend six courses per semester.

Heim 2000: **comparative** ambiguity under modals:

(4)   The draft is 10 pages long. The paper is required to be exactly 5 pages longer than that.
      a.   the paper cannot be longer or shorter than 15 pages
      b.   the minimal length of the paper is 15 pages; it can also be longer than that

(5)   The draft is 10 pages long. The paper is allowed to be less long than that.
      a.   it is possible for the paper to be shorter than the draft
      b.   it is required that the paper be shorter than the draft

Hackl 2000: **comparative numerals** interact with intensional predicates:

(6)   John is required to read **fewer than 6 books**.
      a.   the number of books that John reads is not allowed to exceed 5
      b.   the minimal number of books that John should read is less than 6

Geurts and Nouwen 2007 claim that superlative numerals cannot be so ambiguous, but...
Büring 2008: **superlative numerals are ambiguous in modal contexts**:

(7)   John has to read **at least 3 books**.
      a.   It has to be the case that John reads three or more books.   authoritative
      b.   The speaker is sure that John has to read 3 books.   speaker insecurity

Oda 2008, Beck 2009: ***exactly*-NPs can have an *at least* reading** under modals:

(8)   You are allowed to write exactly 5 pages.
      a.   writing exactly 5 pages is permitted (but you can write more than that, too)
      b.   the maximum you are allowed to write is exactly 5 pages

Solutions have preferentially been specific to the empirical focus of each study.

One notable exception is Krasikova 2010, unifying Heim and Rullmann ambiguities with a pragmatic account based on Fox and Hackl 2006

This work: a variation on Krasikova's and Beck's accounts, deriving the strong reading from a combination of scalar alternatives and exhaustification

1.   THE EXHAUSTIVE READING OF MODAL CONTENT

Main empirical point: the minimal and maximal ambiguity of modal content does not depend on the presence of a comparative or a numeral:

(9) a. If you're not careful, you can scratch the surface.   minimal
     b. If you apply a lot of pressure, you can scratch the surface.   maximal

(10) a. To ensure the right level of teaching we must hire an ASSOCIATE professor. minimal
     b. To bring down the costs we must hire an ASSOCIATE professor.   maximal

(11) a. You can fail the course if you don't start working.   minimal
     b. You can fail the course and still get your degree.   maximal

The minimal reading is what is expected: (12a) doesn't exclude (12b), yielding the minimal interpretation for (12); the same is true for the existential modal in (13).

(12) You must end up with three cards [if you have any fewer, you have cheated]. $\forall$, minimal
     a. $\forall w \in Acc$ [you end up with three cards]
     b. $\exists w \in Acc$ [you end up with four cards] $\rightarrow$ not excluded

(13) You can earn \$50 [even if you have only passed one test]. $\exists$, minimal
     a. $\exists w \in Acc$ [you earn \$50]
     b. $\exists w \in Acc$ [you earn \$51] $\rightarrow$ not excluded

Hypothesis: the additional readings arise from the presence of the silent exhaustive operator EXACT (Beck 2009; for similar lexical items see Krifka 1999, Fox and Hackl 2006, Chierchia 2006 and Chierchia, Fox and Spector to appear), excluding all non-entailed alternatives to the ordinary value of the clause:

(14) EXACT $(p, A) = 1$ if and only if p & $\forall q \in A$ $[\neg (p \rightarrow q) \rightarrow \neg q]$,
     where A is the set of relevant alternatives to p

The alternatives may be introduced by focus or result from Horn scales inherent in cardinals.
note: the maximal reading of "You can have one more cookie" doesn't preclude the child from breathing! Does this commit us to scalar alternatives?

Confirmation: the placement of focus may change the truth-conditions:

(15) a. [Pure white is too flashy] you can rent a CRYSTAL diamond (but not Wesselton).
     b. [We are not millionaires] you can RENT a crystal diamond (but not buy one).

(15a) but not (15b) is compatible with buying a crystal diamond (not an alternative for (15a))

The wide scope of EXACT is independently motivated (= scalar implicature). Matters are more complicated for its narrow scope.

1.1. The wide scope of EXACT

With a **necessity** modal, the alternatives excluded by EXACT are those where the requirement is stronger than what is stated:

(16) You must end up with three cards [that's the only constraint]. $\forall$, minimal/focus
     a. EXACT $[\Box$ [you end up with three cards]]
     b. $\forall w \in Acc$ [you end up with three cards] &
         $\neg[\forall w \in Acc$ [you end up with four cards] &
         $\neg[\forall w \in Acc$ [you end up with five cards]...

This is still **a minimal reading** (= exceeding the requirement is allowed). It is also the default interpretation of a necessity modal.

Extra evidence: the authoritative reading of superlative modified numerals (Büring 2008):
Note: Büring derives this reading by moving the superlative numeral itself (excluded on syntactic grounds)

(17) a. The password must be at least five characters long. authoritative
     b. To become a member of this club, you must pay at least $200,000. speaker insecurity

Further motivation: Nouwen 2008: **negated comparatives** in English express equality:

(18) No more than 30 people showed up.
     a. 30 people showed up
     b. this is unexpectedly little

Simple combination of negation with a comparative (19) yields too weak a reading (20a); to get the preferred reading of (18) non-entailed alternatives should be excluded, yielding (20b):

(19) $[\![$ not more than $]\!] = \lambda P_{\langle d, t \rangle} . \neg [\max_d (P(d)) > \alpha]$

(20) a. $\max_d$ (d-many people showed up) ≤ 30
     b. $\max_d$ (d-many people showed up) $= 30$

If exhaustification is independently necessary for negated comparatives, there's no reason not to use it elsewhere.

If the EXACT operator outscopes a **possibility** modal, the alternatives excluded by the speaker are those where more/other than what is stated is permitted:

(21) You can earn $50 [but only if you work really hard]. ∃, **maximal**
     a. EXACT $[\lozenge[$you earn $50]$
     b. $\exists w \in$ Acc [you earn $50] &
         $\neg \exists w \in$ Acc [you earn $51] & $\neg \exists w \in$ Acc [you earn $52]...

Rendering the permission maximal via exhaustivity also works for *few*:

(22) You are allowed to invite few people.
     a. $\exists w \in$ Acc [you invite few people] minimal
     b. $\exists w \in$ Acc [you invite few people] maximal
         & $\neg \exists w \in$Acc [you invite more than few people]

...and also gives rise to Büring's *third construal* (Büring 2009) -- see below:
Note: Büring derives this reading by moving only the numeral (excluded on syntactic grounds)

(23) a. Kim may have at most three drinks.
     b. The meat you use may have at most 5% of fat.

1.2. Narrow scope of EXACT

Scoping the EXACT operator below a **necessity** modal gives rise to the maximal reading of the modal content: for every possible world the alternatives where more is done than stated are excluded:

(24) You must end up with three cards [one of which is an ace]. ∀, **maximal**
     a. □ [EXACT [you end up with three cards]]
     b. $\forall w \in$ Acc [you end up with three cards] &
         $\neg$[you end up with four cards] &
         $\neg$[you end up with five cards]...

If the EXACT operator takes scope below a **possibility** modal (25a), a very weak condition is excluded (25b): it is asserted that it is possible to do nothing other than what is allowed by the ordinary value of the sentence. This reading can be distinguished from (13) in the context of a lottery when discussing the potential gain from purchasing $100 ticket:

(25) You can earn $50 [but you can also earn $1000].	∃, minimal/focus = (13)
   a.	◊[EXACT [you earn $50]
   b.	∃w∈Acc [[you earn $50] & ¬[you earn $51] & ¬[you earn $52]…]

Why do we have doubts about the narrow scope of EXACT?

   ➢	The maximal reading of the modal content is not as easy to obtain under necessity modals as it is under possibility modals; in fact, it requires so much context that the question arises if it is not the context itself that is providing the upper bound

   ➢	The existence of embedded implicatures is under debate (Chierchia 2006, Chemla and Spector to appear, Chierchia, Fox and Spector to appear vs. Breheny, Katsos and Williams 2006, Geurts 2009, Geurts et al. 2010, etc.)

   ➢	On the other hand, if the "exactly" interpretation of numerically quantified NPs is the default one (Breheny 2008), why is this reading so hard to get?

"More work is required"...

## 2.	EXACTLY (ODA 2008, BECK 2009)

Krifka 1999: modification of cardinal-containing NPs with *at least*, *more than*, *exactly*, etc., operates on scalar alternatives to the cardinal

Scalar alternatives to the cardinal should therefore be invisible!

Beck 2009: *exactly* is not interpreted; the alternatives to the cardinal are captured by EXACT above or below the modal:

(26) You are required to write exactly 5 pages.
   a.	EXACT [∀w ∈ Acc [you write ~~exactly~~ [5]$_F$ pages]]
   b.	∀w ∈ Acc [EXACT [you write ~~exactly~~ [5]$_F$ pages]]

However, nothing prevents the **entire modified numeral** (or even just the modifier) from introducing alternatives under focus: *at least*, *exactly*, *more than*, etc., are all alternatives to each other

Note: Krifka 1999 observes that stressing the cardinal is not necessary, so multiple foci are not required

Confirmation: the focal stress falls on the *exactly*-NP rather than the cardinal

## 3.	INTERMEDIATE CONCLUSION

The presence of the minimal/maximal ambiguity of an expression in the scope of a modal has been used as an argument for QRing that expression. However, some such ambiguities can be derived on independent grounds:

   ➢	for possibility modals stronger alternatives can be excluded by scalar implicature (however it is achieved), giving rise to the minimal/maximal ambiguity

   ➢	for necessity modals the minimal reading can be achieved by an embedded scalar implicature. And if numerically quantified NPs can have the "exactly" reading by virtue of their semantics (Breheny 2008), examples containing measure phrases say nothing about the scope of comparatives

   ➢	the testing ground is thus limited to a downward-entailing (or non-monotonic!) quantifier or degree operator embedded under a universal modal

Potential alternative (cf. Kennedy 2010): the minimal/maximal ambiguity of an expression is always due to QR of that expression. To check this hypothesis we need to consider whether **the maximal/minimal ambiguity of NPs correlates with the *de re/de dicto* interpretation**:

(27) By the end of the game you must hold **a pair of knaves**...
   a.   if you also have a queen, you get six points extra.   minimal
   b.   if they're both black, you get six points extra.   maximal

The wide scope of an NP should yield a *de re* interpretation.

4.   DOWNWARD -ENTAILING DETERMINERS AND DEGREE MORPHEMES

No problems arise with an inferiority comparative or a downward-entailing determiner in the scope of the possibility modal:

(28) a.   You can invite few people.   ✓minimal, ✓maximal
   b.   The paper can be less long than that.   ✓minimal, ✓maximal

Adding EXACT turns the minimal reading into a maximal one:

(29) You are allowed to invite few people.
   a. $\exists w \in$ Acc [you invite few people]   minimal
   b. $\exists w \in$ Acc [you invite few people]   maximal
           & ¬ $\exists w \in$ Acc [you invite more than few people]

Less-comparatives are derived in the same way.

Assuming the semantics of comparatives as in Heim 2000 derive the ambiguity via the scope of *less*:

(30) The draft is 10 pages long. The paper is allowed to be less long than that.
   a.   *allowed > less*: allowed [[less than that][the paper be d-long]]   minimal
           $\exists w \in$ Acc [max {d: long $_w$ (p, d)} < 10 pages]
               in some possible world compatible with what is allowed the length of the paper is less than 10
               pages.

   b.   *less > allowed*: [less than that] [allowed [the paper be d-long]]   maximal
           max {d: $\exists w \in$ Acc [long $_w$ (p, d)}] < 10 pages
               The maximal degree out of the set of all degrees such that there is a possible world compatible
               with what is allowed where the paper is long to these degrees (i.e., the length of the longest
               paper compatible with what is allowed), is less than 10 pages

The maximal reading of the modal content under necessity modals is usually very difficult to obtain, but with an inferiority comparative or a downward-entailing determiner the maximal reading is actually preferred:

(31) The draft is 10 pages long. The paper is required to be less long than that.
   a.   it is possible for the paper to be longer than the draft   minimal
   b.   it is required that the paper be shorter than the draft   maximal

(32) Jessamine is required to invite few people.
   a.   inviting few people is not prohibited   minimal
   b.   inviting more than a few people is prohibited   maximal

This is not unexpected for downward-entailing determiners as they are known to introduce an upper bound: (33a) but not (33b) is compatible with the situation in (33c):

(33) a.   Jessamine has invited a few people.
   b.   Jessamine has invited few people.
   c.   Jessamine has invited many people.

The maximal reading of the modal content is therefore predicted:

(34) $\forall w \in$ Acc [Jessamine invite few people] & ¬[Jessamine invite more than a few people]

Likewise, *less*-comparatives are predicted to give rise to maximal readings by virtue of the fact that they introduce an upper bound, however this is achieved: (35a) entails (35b):

(35) a. The paper is less long than the draft.
      b. ¬ [The paper is as long as the draft or longer]

Nothing in our theory **predicts the minimal reading for downward-entailing determiners and *less*-comparatives** unless we decompose both *less* and *few* into a negative component that can QR independently and the residue (cf. Heim 2006). We find this quite ugly.

Heim 2000 predicts both readings:

(36) The draft is 10 pages long. The paper is required to be less long than that.

    a. *required > less*: required [[less than that][the paper be d-long]]   maximal
$\exists w \in$ Acc [max {d: long $_w$ (p, d)} < 10 pages]
>   in every possible world compatible with what is required the length of the paper is less than 10 pages.

    b. *less > required*: [less than that] [required [the paper be d-long]]   minimal
max {d: $\forall w \in$ Acc [long $_w$ (p, d)}] < 10 pages
>   The maximal degree out of the set of all degrees such that there is a possible world compatible with what is required where the paper is long to these degrees (i.e., the length of the shortest paper compatible with what is required), is less than 10 pages

…but not for downward-entailing quantifiers, unless they are **scalar** (cf. Hackl 2000), contain a degree operator (POS) and can be **decomposed** to yield an independently scoping negation (cf. Heim 2006 for *little*)

Kennedy 2010 might be able to derive it if *few* is assumed to be a number, like *five*:

(37) max {n: $\forall w \in$ Acc [Jessamine invite n people}] = few

Unexpectedly, with the functional modal *must* the minimal reading is either very difficult to obtain or absent altogether:

(38) a. Jessamine must invite few people.
      b. The paper must be less long than that.

Could this be somehow related to the correlation between the maximal/minimal reading of a modal-containing comparative clause and the scope splitting properties of that modal (Heim 1999, Schwarzschild and Wilkinson 2002, Schwarzschild 2004) (Benjamin Spector, p.c.)?

Suggesting that downward-entailing NPs in the sentence-final position may have wide scope as a result of overt movement would give us different truth-conditions:

(39) You are required to invite few people: the Pope, Madonna, and their significant others.

The minimal reading therefore remains **a mystery for an exhaustivity-based view**.

5. EXTENSION: ENCODING THE SPEAKER KNOWLEDGE (BÜRING 2008)

In modal contexts superlative numerals give rise to an ambiguity reflecting how informed the speaker is:

(40) John has to read at least 3 books.
    a. It has to be the case that John reads three or more books.   authoritative
    b. The speaker is sure that John has to read 3 books.   speaker insecurity

Büring 2008: in the **authoritative** reading *at least three* scopes **below** the modal while in the **speaker insecurity** reading it scopes **above** it:

(41) a.　John has to [[at least three] [λd [t $_{John}$ read d-many books]]]　　authoritative
　　b.　$\Box$ [3 = max(λd.John reads d-many books) $\lor$
　　　3 < max(λd.John reads d-many books)]
　　'in every permitted world, the maximum number of books John reads is greater than or equal to 3'

(42) a.　[at least three] [λd [John has to [t $_{John}$ read d-many books]]]　　speaker insecurity
　　b.　3 = max(λd. $\Box$[John reads d-many books]) $\lor$
　　　3 < max(λd. $\Box$ [John reads d-many books])
　　'the highest number *n* s.t. in every permitted world, John reads that n (or more) books is greater than or equal to three'

In order to derive the two scopal possibilities *at least 3* must be a phrasal constituent (an XP).

Island-sensitivity is also incorrectly predicted:

(43)　They must hire three assistant professors and at least one associate professor.
　　a.　authoritative reading: the requirement is to hire at least one associate professor
　　b.　speaker insecurity reading: I don't know how many associate professors they have to hire, but it's no fewer than one

In our proposal the speaker insecurity reading corresponds to the absence of covert operators; inserting EXACT and therefore forcing maximal informativeness results in the authoritative reading for necessity modals:

(44)　John has to read at least 3 books.
　　a.　EXACT [$\Box$ [John reads at least 3 books]]
　　b.　$\forall$w$\in$Acc [John reads 3 books] & $\neg$[$\forall$w $\in$ Acc [John reads more than 3 books]]

With possibility modals adding *at least* renders the maximal reading incoherent (see Büring 2009). Adding *at most* allows us to distinguish two readings: the speaker uncertainty reading (corresponding to the absence of covert operators) and the "third construal" (Büring 2009):
Presupposition: *at most*, like *few*, is downward-entailing (see below)

(45)　They can stay underwater for at most 10 minutes.
　　a.　EXACT [$\Diamond$[they stay underwater for at most 10 minutes]
　　b.　$\exists$w$\in$Acc [they stay underwater for at most 10 minutes] &
　　　$\neg\exists$w $\in$ Acc [they stay underwater for more than 10 minutes]
　　c.　the maximal time that they can stay underwater is 10 minutes　　"third construal"

With necessity modals *at most* gives rise to a speaker insecurity reading and an authoritative reading:

(46) a.　He had to wait at most three months.
　　b.　The meat you use must have at most 5% of fat.

The authoritative reading can be derived from the speaker insecurity reading as in (44).

6.　STRUCTURAL APPROACHES TO THE MINIMAL/MAXIMAL AMBIGUITY WITH MODALS

Hallmark: modal content ambiguity is due to the scope of a syntactic generalized quantifier

6.1.　Heim 2000: QR of degree morphemes

Heim 2000 uses the maximal/minimal ambiguity of comparatives under modals to argue that **degree operators** interact with intensional predicates and are therefore syntactic quantifiers:

(47)　$[\![$more$]\!]$= λg$\in$ D $_{\langle d, t\rangle}$. λf $\in$ D $_{\langle d, t\rangle}$. max (f) > max (g)
　　where max (P) = ιd $\in$ D$_d$. P(d) = 1 and $\forall$d$'$ $\in$ D$_d$ [P(d$'$) = 1 $\rightarrow$ d$'$ $\leq$ d]

Kennedy 1999: if the degree operator moves, it should be able to scope over other operators. Such readings are in fact not attested (Heim 2000):

(48) John is 4 feet tall. Every girl is exactly 1 inch taller than that.
    a. $\forall x$ [girl(x) $\rightarrow$ max {d: tall(x,d)} = 4′ + 1″]
    b. *max {d: $\forall x$ [girl(x) $\rightarrow$ tall(x,d)}] = 4′ + 1″   the shortest girl is 1″ taller than John

The reading in (48b), where the comparative scopes over the universal, is not available

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(49) Kennedy's generalization (Heim 2000):

If the scope of a quantificational DP contains the trace of a degree operator, it also contains that degree operator itself.

However, degree operators do interact with **intensional predicates** (Heim 2000); therefore, degree operators can scope, as shown by the ambiguity of (50):

(50) The draft is 10 pages long. The paper is required to be exactly 5 pages longer than that.
    a.   *required > -er*: required [[exactly 5 pages -er than that][the paper be d-long]]
        $\forall w \in$ Acc [max {d: long $_w$ (p, d)} = 15 pages]
            in every possible world compatible with what is required the maximal length of the paper is
            exactly 5 pages more than 10 pages.   = the *exact* length of the paper is 15 pages

    b.   *-er > required*: [exactly 5 pages -er than that] [required [the paper be d-long]]
        max {d: $\forall w \in$ Acc [long $_w$ (p, d)}] = 15 pages
            As before, we construct the set of all degrees such that the paper is long to these degrees in
            any possible world compatible with what is required, and then take the maximal such degree.
            This maximal degree, corresponding to the length of the shortest paper compatible with what
            is required, has to be 5+10 pages.   = the *minimal* length of the paper is 15 pages

Hackl 2000: since comparative numerals involve comparative morphemes, they interact with intensional predicates exactly as comparatives do:

(51) John is required to read **fewer than 6 books**.
    a.   [require [[fewer than 6] [λn [John reads [[n many] books]]]]]
            in every possible world compatible with what is required the maximal number of books that
            John reads is less than six: John shouldn't read more than 5 books.

    b.   [[fewer than 6] [λn [require [John reads [[n many] books]]]]]
            the maximal number such that John reads that number of books in every possible world (i.e.,
            the minimal number of books that John should read) is less than 6

Morphosyntactic evidence rules against this analysis: the lexical integrity hypothesis (parts of words don't move) + some evidence that synthetic comparatives involve head-movement and therefore cannot form a maximal projection to the exclusion of the adjective (Matushansky 2011)

Also, this analysis does not extend to the minimal/maximal ambiguity of NPs under modals (Oda 2008, Beck 2009, Kennedy 2010, this work)

The analysis proposed by Hackl 2000 also requires the presence of a silent *many* in every NP containing the cardinal.

6.2. Oda 2008: QR of exactly-NPs

Oda 2008, Beck 2009: NPs containing *exactly* can give rise to both a minimal and a maximal reading in the scope of a modal

Oda 2008 according to Beck 2009: *exactly*-NPs are themselves degree operators:

(52) ⟦exactly five pages⟧ = λD . max (D) = 5 pages

... and can QR:

(53) You are allowed to write exactly 5 pages.
  a. allowed [[exactly 5 pages] [λd . [you write d-much]]]   minimal
      ∃w ∈ Acc [max {d: you write d-much} = 5 pages]
  b. [[exactly 5 pages] [λd . allowed [you write d-much]]]   maximal
      max {d: ∃w ∈ Acc [you write d-much]} = 5 pages

(54) You are required to write exactly 5 pages.
  a. required [[exactly 5 pages] [λd . [you write d-much]]]   maximal
      ∀w ∈ Acc [max {d: you write d-much} = 5 pages]
  b. [[exactly 5 pages] [λd . required [you write d-much]]]   minimal
      max {d: ∀w ∈ Acc [you write d-much]} = 5 pages

This analysis (unlike that due to Beck 2009 alone, formulated in the terms of alternatives and EXACT) does not extend to NPs not containing *exactly*.

Given that one doesn't write degrees or quantities, further assumptions are necessary.

It also suffers from the same syntactic problems as Kennedy's analysis

6.3. Kennedy 2010: the scope of numerals

Kennedy 2010, see also Rothstein 2011: number words are scope-taking degree quantifiers:

(55) ⟦seven⟧ = λP ∈ D$_{\langle d, t \rangle}$. max {m : P (m)} = 7

Kennedy's motivation: interpretation of sentences containing *average* (Kennedy and Stanley 2009: mathematical computation is part of the lexical entry for *average*)

The maximal and minimal readings of a cardinal-containing NP under a modal therefore arise as a result of the different scope positions of the number word:

(56) a.
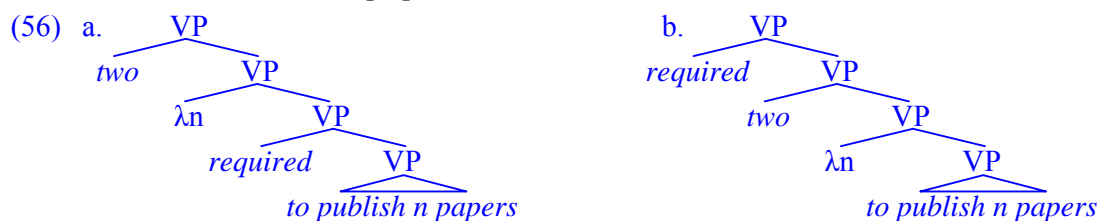
Unlike in Rothstein's approach, the number word cannot be interpreted *in situ*, since its sister, the NP, is generally not assumed to have the semantic type $\langle d, t \rangle$.

Problem: though referential DPs and coordinate structures are islands for QR (Ruys 1992), they can nonetheless contain number words:

(57) a. those two books
  b. the eight planets of the Solar System
  c. two specific girls

Needless to say, cardinals are incorrectly predicted to be sensitive to other islands as well.

Unattested interpretations are predicted with number words scoping higher then quantifiers:

(58) a. No one read five books from this list.
  b. max {n : ¬∃x [x read n books]} = 5

...which are, however, ruled out by the Heim-Kennedy generalization.

Predictions for modified numerals are unclear.

7.   PRAGMATIC APPROACHES TO THE MINIMAL/MAXIMAL AMBIGUITY WITH MODALS

Hallmark: exhaustification
Note: Breakstone et al. to appear argue against pragmatic approaches to the minimal/maximal ambiguity with modals.
Kennedy 2010: psycholinguistic evidence against deriving exact readings of cardinals via scalar implicature has been advanced by Geurts et al. 2010, Huang and Snedeker 2009, Musolino 2004, Noveck 2001, Papafragou and Musolino 2003

Pragmatic proposals don't predict the behavior of comparatives in ellipsis and ACD licensing (Heim 2000)

7.1.   Beck 2009: alternatives and exhaustification

Beck 2009 only deals with NPs containing *exactly* and *less*-comparatives. For the former she suggests that *exactly* introduces alternatives but is not itself interpreted. These alternatives are evaluated by EXACT (which Beck calls EXACT).

(59)   You are required to write exactly 5 pages.
    a.   EXACT $[\forall w \in$ Acc [you write ~~exactly~~ [5 pages]$_F$]]
    b. $\forall w \in$ Acc [EXACT [you write ~~exactly~~ [5 pages]$_F$]]

Heim 2000: *less*-comparatives also show scope ambiguities under modals:

(60)   [This draft is 10 pages long.] The paper is required to be less long than that.
    a.   The minimum length required for the paper is less than 10 pages.   less > required
    b.   Papers longer than 10 pages are not allowed.   required > less

Beck 2009: the negative degree morpheme *little* (cf. Heim 2006) is interpreted as *much*, but it also introduces focus alternatives, which are evaluated by the higher covert operator AT MOST.

Problems:

➢   No connection between the modifier (*exactly*, *little*) and the alternative-evaluating operator predicts a mix-and-match generation

➢   Island violations and non-local configurations are predicted

➢   The availability of *precisely*, alongside *exactly*, is not expected, nor is the lexical nature of such modifiers

This proposal forms the basis for ours, but is insufficiently general and introduces a stipulated focus-evaluating operator for *less*-comparatives.

7.2.   Krasikova 2010: free choice implicature plus exhaustification

Krasikova 2010 unifies Heim and Rullmann ambiguities on the basis of Fox and Hackl 2006: the two readings do not depend on comparatives:

(61)   a.   You are allowed to arrive at 10 PM.
    b.   You are allowed to arrive earlier than 10 PM.
    c.   You are allowed to arrive before 10 PM.

Maximal reading in the context of the question "When are we allowed to arrive?": providing the latest permissible time of arrival.

Note: this is precisely what is expected under our proposal: a wh-question puts focus on the scalar term and the answer has to be exhaustive

Minimal reading arises in the context of an alternative question.

Fox and Hackl 2006: the strong reading of (61) depends on two pragmatic mechanisms:

- ➢ free choice implicature: if some time before *t* is permissible, any time before *t* is permissible
- ➢ scalar implicature: the statement actually made is the maximally informative one

The latter is achieved by a covert operator:

(62) $[\![exh]\!] (A)(p) = \lambda w \, p(w) \, \& \, \forall q \in \text{I-E}(p, A) \, \neg q(w)$, where
the set of innocently excludable alternatives I-E $(p, A) = \cap \{A' \subseteq A \mid A'$ is a maximal
set in A, s.t. $\{\neg r : r \in A'\} \cup \{p\}$ is consistent$\}$

To derive the Heim ambiguity with *less* the operator *exh* is applied recursively.

To derive the Heim ambiguity with *exactly*-differentials the meaning of *exactly* is somehow shifted to *at most*, which allows the free choice implicature that is then strengthened by *exh*

To derive the Rullmann ambiguity (for both inferiority and superiority comparatives) three rounds of exhaustification local to the degree clause, plus the *at least*/*at most* ambiguity of a numeral are required.

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